EEE2035F: Signals and Systems I

Class Test 1

16 March 2015

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totaling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

---

1. (5 marks) Suppose

$$p_\tau(t) = \begin{cases} 1 & |t| \leq \tau/2 \\ 0 & \text{otherwise} \end{cases}$$

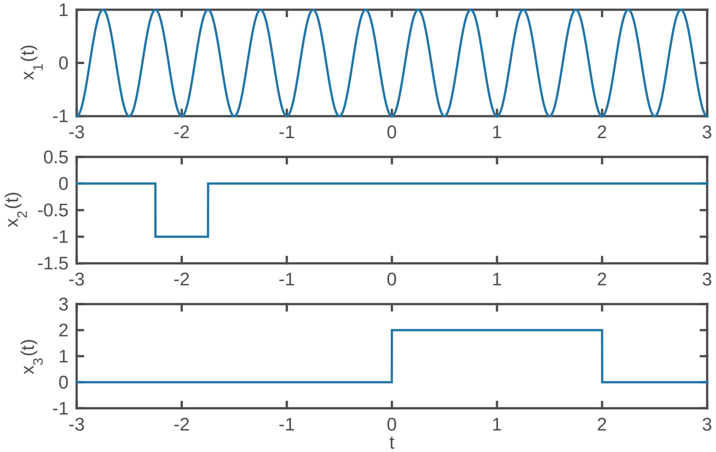
and $u(t)$ is the unit step. Sketch the following signals over the range $-3 \leq t \leq 3$:

(a) $x_1(t) = \cos(4\pi t - \pi)$

(b) $x_2(t) = -p_1(2t + 4)$

(c) $x_3(t) = 2u(t)u(2 - t)$.

Plots as follows:



---

2. (5 marks) Suppose the output $y(t)$ of a system is related to the input $x(t)$ via the relationship
$$y(t) = x(2t),$$
and $u(t)$ is the unit step.

(a) Plot the output when the input is $x(t) = u(t)$.

(b) Plot the output when the input is $x(t) = u(t-1)$.

(c) Is the system time invariant? Why?

(d) Show that the system is linear.

(a) For input $x_1(t) = u(t)$ the output is $y_1(t) = x_1(2t) = u(2t) = u(t)$. (Plot the unit step.)

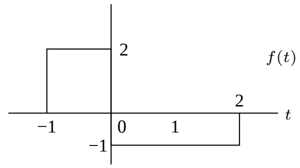
(b) For input $x_2(t) = u(t-1)$ the output is
$$y_2(t) = x_2(2t) = u(2t-1) = u(2(t-1/2)) = u(t-1/2).$$
(Plot the unit step shifted to the right by 1/2.)

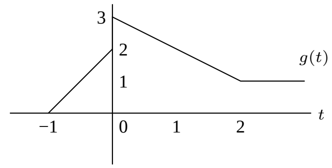
(c) From the previous two questions, the input $x_1(t) = u(t)$ produces the output $y_1(t) = u(t)$, and the input $x_2(t) = u(t-1)$ produces the output $y_2(t) = u(t-1/2)$. Since $x_2(t) = x_1(t-1)$ for a time invariant system we should have $y_2(t) = y_1(t-1)$. This is not the case so the system is not time invariant.

(d) For two arbitrary signals $x_1(t)$ and $x_2(t)$ the following input-output pairs are valid:
$$x_1(t) \longrightarrow y_1(t) = x_1(2t)$$
$$x_2(t) \longrightarrow y_2(t) = x_2(2t).$$
Now consider the input $x(t) = ax_1(t) + bx_2(t)$. The output will be
$$y(t) = x(2t) = ax_1(2t) + bx_2(2t) = ay_1(t) + by_2(t).$$
Therefore, for any a and b we see that the input-output pair
$$ax_1(t) + bx_2(t) \longrightarrow ay_1(t) + by_2(t)$$
is always valid, and the system is therefore linear.
(You could also show that homogeneity and additivity each hold separately.)

---

3. (5 marks) Suppose we have the following signals:



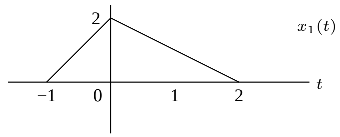


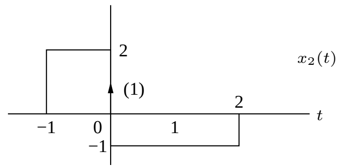
Plot the following:

(a) $x_1(t) = \int_{-\infty}^{t} f(\lambda) d\lambda$

(b) $x_2(t) = \frac{d}{dt} g(t)$ (the generalised derivative).

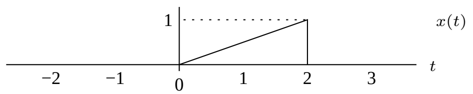
Plots as follows:





---

4. (5 marks) Suppose $s(t)$ is as defined as



Plot the following:

(a) $y_1(t) = s(t) * \delta(t-1)$

(b) $y_2(t) = s(-t) * \delta(t-1)$

(c) $y_3(t) = s(t)\delta(t-1)$.

(d) $y_4(t) = s(-t)\delta(t-1)$.

Plots as follows: