

EEE2035F: Signals and Systems I

Class Test 1

16 March 2015

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
 - This test has *four* questions, totaling 20 marks.
 - Answer *all* the questions.
 - You have 45 minutes.
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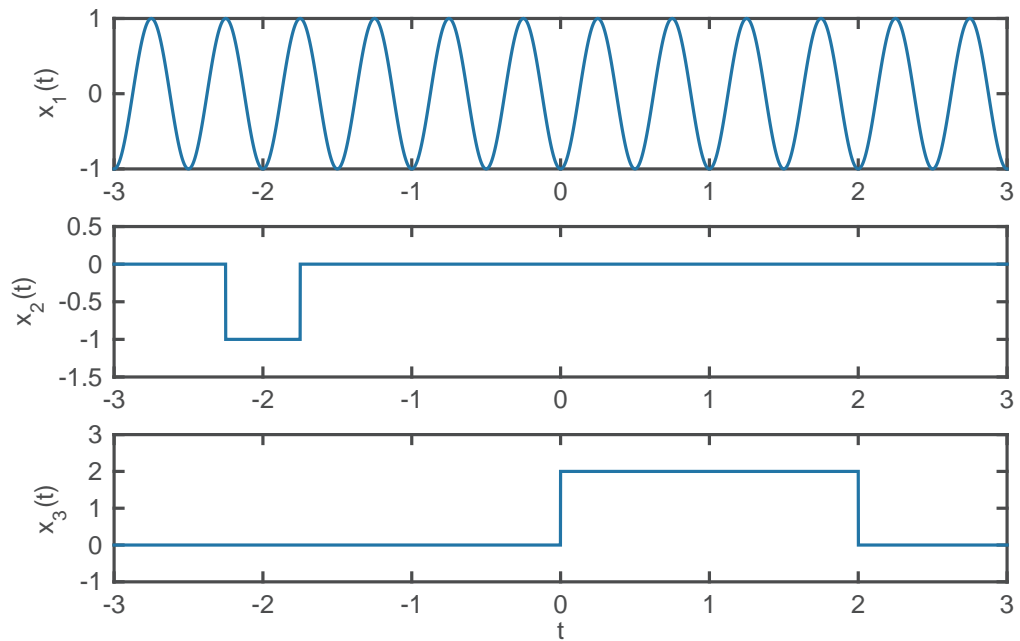
1. (5 marks) Suppose

$$p_\tau(t) = \begin{cases} 1 & |t| \leq \tau/2 \\ 0 & \text{otherwise} \end{cases}$$

and $u(t)$ is the unit step. Sketch the following signals over the range $-3 \leq t \leq 3$:

- (a) $x_1(t) = \cos(4\pi t - \pi)$
- (b) $x_2(t) = -p_1(2t + 4)$
- (c) $x_3(t) = 2u(t)u(2 - t)$.

Plots as follows:



2. (5 marks) Suppose the output $y(t)$ of a system is related to the input $x(t)$ via the relationship

$$y(t) = x(2t),$$

and $u(t)$ is the unit step.

- (a) Plot the output when the input is $x(t) = u(t)$.
- (b) Plot the output when the input is $x(t) = u(t - 1)$.
- (c) Is the system time invariant? Why?
- (d) Show that the system is linear.

- (a) For input $x_1(t) = u(t)$ the output is $y_1(t) = x_1(2t) = u(2t) = u(t)$. (Plot the unit step.)
- (b) For input $x_2(t) = u(t - 1)$ the output is

$$y_2(t) = x_2(2t) = u(2t - 1) = u(2(t - 1/2)) = u(t - 1/2).$$

(Plot the unit step shifted to the right by $1/2$.)

- (c) From the previous two questions, the input $x_1(t) = u(t)$ produces the output $y_1(t) = u(t)$, and the input $x_2(t) = u(t - 1)$ produces the output $y_2(t) = u(t - 1/2)$. Since $x_2(t) = x_1(t - 1)$ for a time invariant system we should have $y_2(t) = y_1(t - 1)$. This is not the case so the system is not time invariant.
- (d) For two arbitrary signals $x_1(t)$ and $x_2(t)$ the following input-output pairs are valid:

$$\begin{aligned}x_1(t) &\longrightarrow y_1(t) = x_1(2t) \\x_2(t) &\longrightarrow y_2(t) = x_2(2t).\end{aligned}$$

Now consider the input $x(t) = ax_1(t) + bx_2(t)$. The output will be

$$y(t) = x(2t) = ax_1(2t) + bx_2(2t) = ay_1(t) + by_2(t).$$

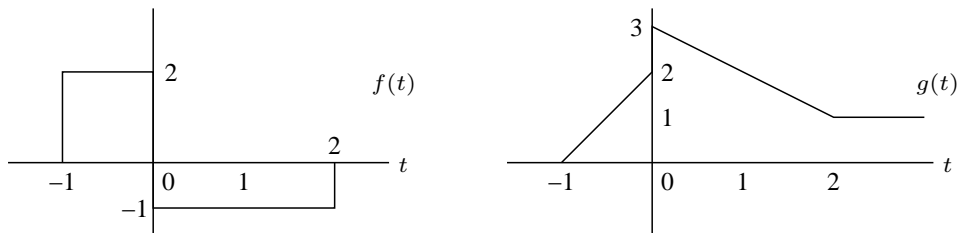
Therefore, for any a and b we see that the input-output pair

$$ax_1(t) + bx_2(t) \longrightarrow ay_1(t) + by_2(t)$$

is always valid, and the system is therefore linear.

(You could also show that homogeneity and additivity each hold separately.)

3. (5 marks) Suppose we have the following signals:

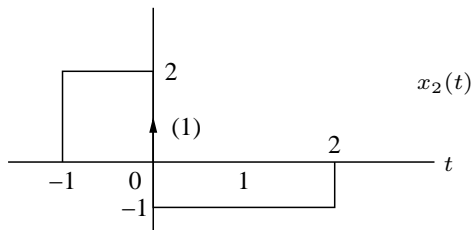
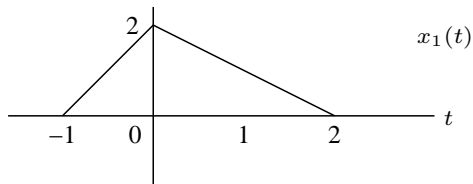


Plot the following:

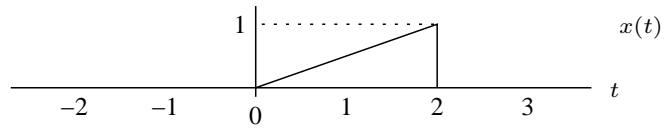
(a) $x_1(t) = \int_{-\infty}^t f(\lambda) d\lambda$

(b) $x_2(t) = \frac{d}{dt}g(t)$ (the generalised derivative).

Plots as follows:



4. (5 marks) Suppose $s(t)$ is as defined as



Plot the following:

- (a) $y_1(t) = s(t) * \delta(t - 1)$
- (b) $y_2(t) = s(-t) * \delta(t - 1)$
- (c) $y_3(t) = s(t)\delta(t - 1)$.
- (d) $y_4(t) = s(-t)\delta(t - 1)$.

Plots as follows:

