EEE2035F: Signals and Systems I

Class Test 1

11 March 2013

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totaling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

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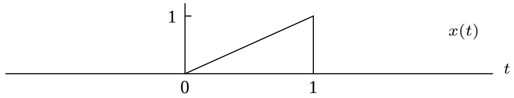
1. (5 marks) Suppose $x(t)$ is as shown below:



Sketch the following signals:

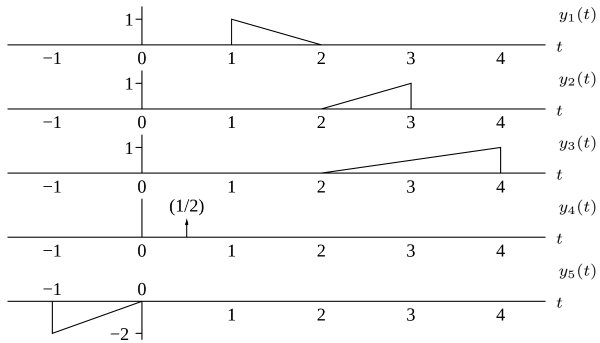
(a) $y_1(t) = x(2-t)$

(b) $y_2(t) = x(t-2)$

(c) $y_3(t) = x(t/2-1)$

(d) $y_4(t) = x(t)\delta(t-1/2)$

(e) $y_5(t) = -2x(-t)$.



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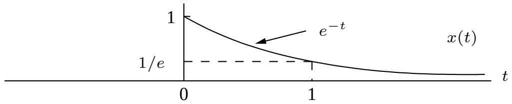
2. (5 marks) The signal $x(t) = e^{-t}u(t)$ is shown below:



Find and sketch the following:

(a) $y(t) = \frac{d}{dt}x(t)$ (the generalised derivative)

(b) $z(t) = \int_{-\infty}^{t} x(\lambda)d\lambda$.

In each case write down a mathematical expression for your answer.

(a) For $t < 0$ the slope is zero, and for $t > 0$ the slope is $\frac{d}{dt}x(t) = -e^{-t}$. The discontinuity of size $+1$ at the origin means that the generalised derivative should have an impulse of size $+1$ at that point. Thus the generalised derivative can be written as

$$y(t) = \frac{d}{dt}x(t) = \delta(t) - e^{-t}u(t),$$

or any equivalent form, and is sketched below.

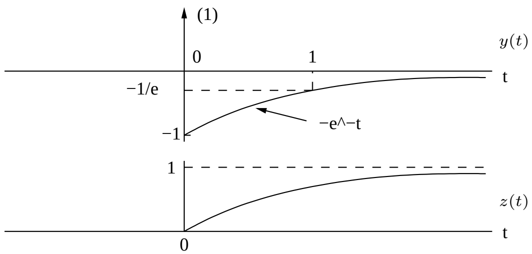
(b) Since $x(\lambda) = 0$ when $\lambda < 0$, for $t < 0$ we must have

$$z(t) = \int_{-\infty}^{t} x(\lambda)d\lambda = \int_{-\infty}^{t} 0 d\lambda = 0.$$

For $t > 0$:

$$z(t) = \int_{-\infty}^{t} e^{-\lambda}u(\lambda)d\lambda = \int_{0}^{t} e^{-\lambda}d\lambda = -[e^{-\lambda}]_{\lambda=0}^{t} = -(e^{-t} - 1) = 1 - e^{-t}.$$

Overall the solution can be written as $z(t) = (1 - e^{-t})u(t)$ or equivalent, and is sketched below.



3. (5 marks) The input $x(t)$ and output $y(t)$ from a system satisfies the relationship

$$y(t) = x(2t).$$

The signal $u(t)$ is the standard unit step.

(a) Find and sketch the output $y_1(t)$ when the input is $x_1(t) = u(t)$

(b) Find and sketch the output $y_2(t)$ when the input is $x_2(t) = u(t-1)$

(c) Based on your answers, does the system appear to be time invariant? Why?

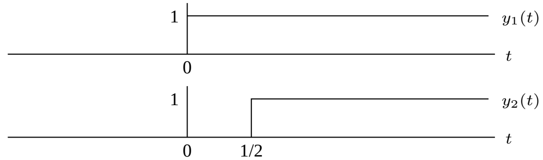
(a) According to the input-output relationship we must have

$$y_1(t) = x_1(2t) = u(2t) = u(t),$$

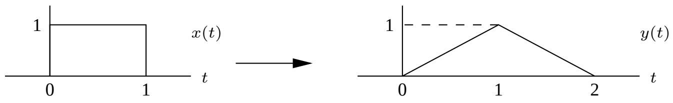
where the last equality holds because compressing the axis around the origin doesn't change the unit step. Sketched below.

(b) The second output satisfies $y_2(t) = x_2(2t) = u(2t-1) = u(2(t-1/2)) = u(t-1/2)$, again because $u(2t) = u(t)$. Sketched below.

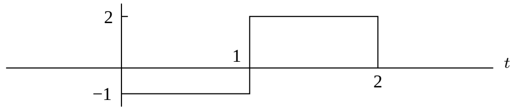
(c) The input $x_2(t)$ is just the input $x_1(t)$ shifted to the right by one unit. If the system were time invariant then $y_2(t)$ would just be $y_1(t)$ shifted to the right by one unit. Since it isn't, the system is not time invariant.



4. (5 marks) For a given linear and time invariant system it is known that the input $x(t)$ and output $y(t)$ below is a valid input-output pair:



Use this information to find the response to the input



By homogeneity from the given pair, $x_1(t) \longrightarrow y_1(t)$ shown below is a valid pair. By homogeneity and time invariance applied to the same pair, $x_2(t) \longrightarrow y_2(t)$ below is a valid pair. Using additivity on these two derived pairs we get the valid input-ouput pair $x_3(t) \longrightarrow y_3(t)$ below, so $y_3(t)$ gives the required answer.