EEE2035F: Signals and Systems I

Class Test 2

23 April 2012

SOLUTIONS

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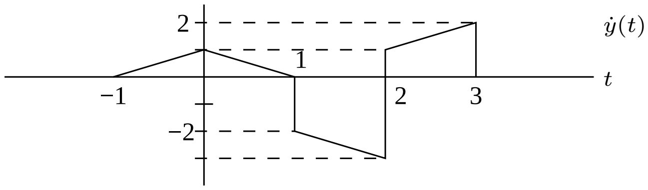
1. From the properties of convolution we have $\dot{y}(t) = h(t) * \dot{x}(t)$. However, $\dot{x}(t) = 2\delta(t-1) - 4\delta(t) + 2\delta(t-1)$, so

$$\dot{y}(t) = 2h(t+1) - 4h(t) + 2h(t-1).$$

A plot is below:



The signal given is the indefinite integral:

$$y(t) = \int_{-\infty}^{t} \dot{y}(\lambda) d\lambda.$$

The output will be zero prior to $t = -1$, at which point it will start to go positive. Thus $a = -1$. When $t = 1$ the total area to the left is 1, so $c = 1$. When $t = 3$ the total area to the left will be zero, and remains zero for $t > 3$. Thus $b = 3$.

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2. (a) If $u(t) \longrightarrow g(t)$ is a valid input-output pair then from the derivative property of convolution

$$\delta(t) = \frac{d}{dt}u(t) \quad \longrightarrow \quad \frac{d}{dt}g(t)$$

is a valid pair. But the left hand side is just the impulse response, so the right hand side must be the impulse response. Thus $h(t) = \dot{g}(t)$, or

$$g(t) = \int_{-\infty}^{t} h(\lambda) d\lambda.$$

When $t < 0$ we have $g(t) = 0$. For $t \geq 0$ the output is

$$g(t) = \int_0^t \frac{1}{\tau_0} e^{-\frac{1}{\tau_0}\lambda} d\lambda = \frac{1}{\tau_0}\left[-\tau_0 e^{-\frac{1}{\tau_0}\lambda}\right]_{\lambda=0}^{t}$$
$$= \frac{1}{\tau_0}\left(-\tau_0 e^{-\frac{1}{\tau_0}t} + \tau_0\right) = 1 - e^{-\frac{1}{\tau_0}t}.$$

Thus we can write $g(t) = (1 - e^{-\frac{1}{\tau_0}t})u(t)$.

(b) The input can be written as $x(t) = u(t) - 2u(t-1) + u(t-2)$. However, by linearity and time invariance the following are all valid input-output pairs:

$$u(t) \longrightarrow g(t), \quad -2u(t-1) \longrightarrow -2g(t-1), \quad \text{and} \quad u(t-2) \longrightarrow g(t-2).$$

The output is therefore

$$y(t) = g(t) - 2g(t-1) + g(t-2)$$
$$= (1 - e^{-\frac{1}{\tau_0}t})u(t) - 2(1 - e^{-\frac{1}{\tau_0}(t-1)})u(t-1) + (1 - e^{-\frac{1}{\tau_0}(t-2)})u(t-2).$$

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3. (a) The Fourier series representation of the signal is

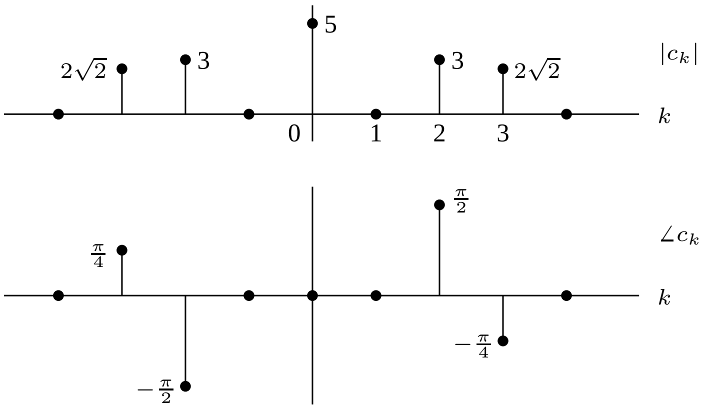
$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}.$$

By inspection we observe that the fundamental frequency is $\omega_0 = 1$.

(b) The coefficients can also be found by inspection:

$$c_0 = 5, \quad c_2 = c_{-2}^* = 3j = 3e^{j\frac{\pi}{2}}, \quad \text{and} \quad c_3 = c_{-3}^* = 2 - 2j = 2\sqrt{2}e^{-j\frac{\pi}{4}}$$

with all other coefficients zero. These are plotted below:



(c) We know the signal is real because $c_k = c_{-k}^*$ for all k. (Magnitude is even and phase is odd.)

(d) From Parseval's theorem the total power is

$$P = \sum_{k=-\infty}^{\infty} |c_k|^2 = (2\sqrt{2})^2 + 3^2 + 5^2 + 3^2 + (2\sqrt{2})^2 = 59 \text{ Watts.}$$

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4. (a) The output is

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda = \int_0^1 e^{j\omega_0(t-\lambda)}d\lambda = e^{j\omega_0 t}\left[\frac{-1}{j\omega_0}e^{-j\omega_0\lambda}\right]_{\lambda=0}^{1}$$
$$= \frac{1-e^{-j\omega_0}}{j\omega_0}e^{j\omega_0 t} = H(\omega_0)e^{j\omega_0 t}.$$

(b) The output will be

$$y(t) = H(2\pi)e^{j2\pi t} = \frac{1-e^{-j2\pi}}{j2\pi}e^{j2\pi t} = 0.$$

(c) If the action of the system is denoted by the transformation $T\{\cdot\}$, then by linearity and time invariance we have

$$y(t) = T\{x(t)\} = T\{x(t)\} = T\left\{\sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}\right\} = \sum_{k=-\infty}^{\infty} T\{c_k e^{jk\omega_0 t}\}$$
$$= \sum_{k=-\infty}^{\infty} c_k T\{e^{jk\omega_0 t}\} = \sum_{k=-\infty}^{\infty} c_k H(k\omega_0)e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t}.$$

Therefore

$$d_k = c_k\frac{1-e^{-jk\omega_0}}{jk\omega_0} = c_k\frac{1-e^{-jk\pi}}{jk\pi}.$$
