EEE2035F: Signals and Systems I

Class Test 2

23 April 2012

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

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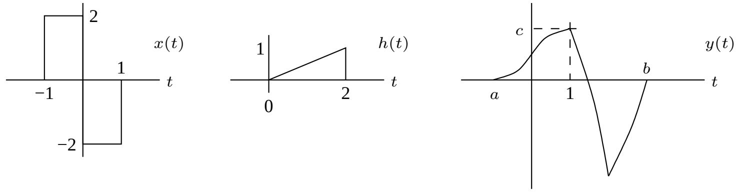
1. (5 marks) For the following signals, $y(t) = h(t) * x(t)$:

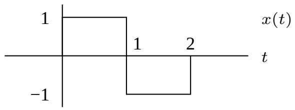


Find the values of a, b, and c, justifying all answers.

2. (5 marks) The impulse response of a LTI system is given by $h(t) = \frac{1}{\tau_0} e^{-t/\tau_0} u(t)$.

(a) Find the step response $g(t)$ of the system, or the response of the system to the unit step input $x(t) = u(t)$.

(b) Find a simple expression for the response $y(t)$ of the system to the input $x(t)$ below:



(Hint: the result from part (a) can be useful here. If you couldn't find the answer for that part, then just express the response in terms of $g(t)$.)

3. (5 marks) Consider the periodic signal $x(t)$ given by the expression

$$x(t) = (2 + 2j)e^{-j3t} - 3je^{-j2t} + 5 + 3je^{j2t} + (2 - 2j)e^{j3t}.$$

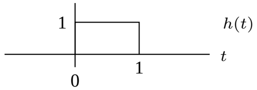
(a) What is the fundamental frequency of $x(t)$?

(b) Sketch the Fourier spectrum (the Fourier series coefficients).

(c) Is $x(t)$ a real signal? Why?

(d) What is the total power of $x(t)$?

4. (5 marks) The impulse response of a LTI system is known to be



(a) Use the definition of convolution to show that when the input is $x(t) = e^{j\omega_0 t}$, the output is $y(t) = H(\omega_0)e^{j\omega_0 t}$ with

$$H(\omega) = \frac{1 - e^{-j\omega}}{j\omega}.$$

(b) Using this result (or otherwise) find the output when the input is $x(t) = e^{j2\pi t}$.

(c) Suppose now that the input is

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t},$$

with $c_0 = 3$, $c_1 = c_{-1}^* = 2$ and $c_2 = c_{-2}^* = 1$, and all other coefficients zero. The output can be expressed as

$$y(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t},$$

Specify the values for d_k.

INFORMATION SHEET

Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t-c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of t	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \ldots$
Frequency shift	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0$ real
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n}x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \ldots$
Integration	$\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Convolution in time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

Common Fourier Transform Pairs

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$	$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$
$1 \quad (-\infty < t < \infty)$	$2\pi\delta(\omega)$
$-0.5 + u(t)$	$\frac{1}{j\omega}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t-c)$	$e^{-j\omega c} \quad$ (c any real number)
$e^{-bt}u(t)$	$\frac{1}{j\omega + b} \quad (b > 0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0) \quad$ (ω_0 any real number)
$p_\tau(t)$	$\tau \text{sinc} \frac{\tau\omega}{2\pi}$
$\tau \text{sinc} \frac{\tau t}{2\pi}$	$2\pi p_\tau(\omega)$
$\left(1 - \frac{2\lvert t\rvert}{\tau}\right) p_\tau(t)$	$\frac{\tau}{2} \text{sinc}^2 \left(\frac{\tau\omega}{4\pi}\right)$
$\frac{\tau}{2} \text{sinc}^2 \frac{\tau t}{4\pi}$	$2\pi \left(1 - \frac{2\lvert\omega\rvert}{\tau}\right) p_\tau(\omega)$
$\cos(\omega_0 t + \theta)$	$\pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$
$\sin(\omega_0 t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$

Trigonometric identities

$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) \qquad \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$