EEE2035F: Signals and Systems I

Class Test 1

11 March 2011

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 25 marks.
- Answer *all* the questions.
- You have 45 minutes.

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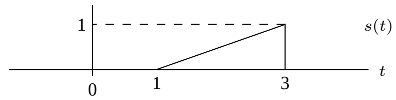
1. (10 marks) Plot the signals given below. Where appropriate, assume that $s(t)$ is the signal



(a) $x_1(t) = \cos(\pi t)$.

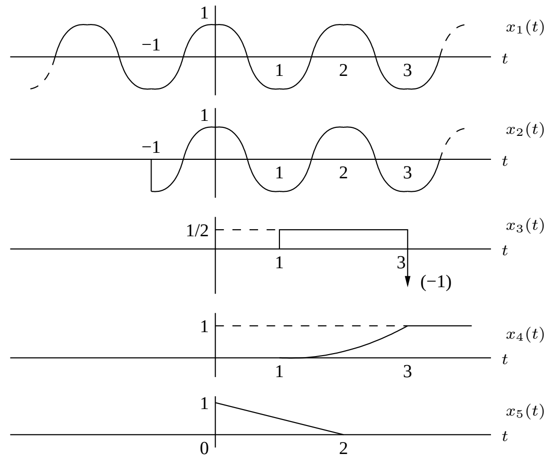
(b) $x_2(t) = \cos(\pi t)u(t+1)$.

(c) $x_3(t) = \frac{d}{dt}s(t)$ (the generalised derivative).

(d) $x_4(t) = \int_{-\infty}^{t} s(\lambda)d\lambda$.

(e) $x_5(t) = s(3-t)$.

Plots as follows:



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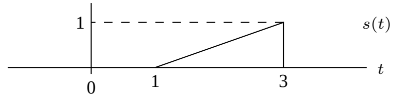
2. (5 marks) Let $s(t)$ be the signal



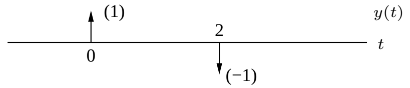
and suppose that $y(t) = \delta(t) - \delta(t-2)$.

(a) Plot $y(t)$.

(b) Calculate $\int_{-\infty}^{\infty} s(t)y(t)dt$.

(c) Find $z(t) = \int_{-\infty}^{\infty} s(\lambda)y(t-\lambda)d\lambda$.

(a) Plot is as follows:



(b) The solution is obtained using the sifting property:

$$\int_{-\infty}^{\infty} s(t)y(t)dt = \int_{-\infty}^{\infty} s(t)[\delta(t) - \delta(t-2)]dt = \int_{-\infty}^{\infty} s(t)\delta(t)dt - \int_{-\infty}^{\infty} s(t)\delta(t-2)dt$$
$$= \int_{-\infty}^{\infty} s(0)\delta(t)dt - \int_{-\infty}^{\infty} s(2)\delta(t-2)dt = s(0) - s(2).$$

(c) Again using the sifting property:

$$\begin{aligned}\int_{-\infty}^{\infty} s(\lambda)y(t-\lambda)d\lambda &= \int_{-\infty}^{\infty} s(\lambda)[\delta(t-\lambda) - \delta(t-\lambda-2)]d\lambda \\ &= \int_{-\infty}^{\infty} s(\lambda)\delta(t-\lambda)d\lambda - \int_{-\infty}^{\infty} s(\lambda)\delta(t-\lambda-2)d\lambda \\ &= \int_{-\infty}^{\infty} s(t)\delta(t-\lambda)d\lambda - \int_{-\infty}^{\infty} s(t-2)\delta(t-\lambda-2)d\lambda \\ &= s(t) - s(t-2).\end{aligned}$$

[too much repetition and complexity here]

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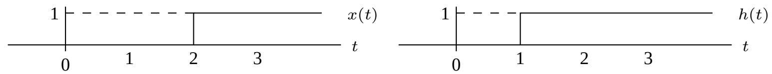
3. (5 marks) Let $x(t) = u(t-2)$ and $h(t) = u(t-1)$, where $u(t)$ is the unit step

$$u(t) = \begin{cases} 1 & (t \geq 0) \\ 0 & (t < 0). \end{cases}$$

(a) Plot $x(t)$ and $h(t)$.

(b) Find and plot $y(t) = x(t) * h(t)$.

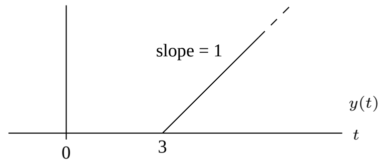
(a) Plots as follows:



(b) Graphically is easier, but analytically the solution is

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda = \int_{-\infty}^{\infty} u(\lambda-2)u(t-\lambda-1)d\lambda$$
$$= \int_{2}^{\infty} u(t-\lambda-1)d\lambda = \int_{2}^{t-1} d\lambda.$$

Thus $y(t) = 0$ if $t - 1 < 2$, or if $t < 3$. For $t \geq 3$ we have $y(t) = t - 1 - 2 = t - 3$:



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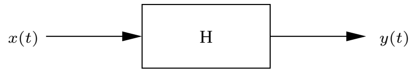
4. (5 marks) Suppose we have a system



that obeys the input-output relationship $y(t) = x(t) + 1$.

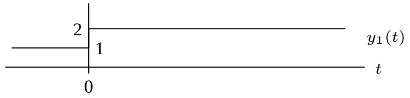
(a) Find and plot the output $y_1(t)$ when the input is $x_1(t) = u(t)$.

(b) Find and plot the output $y_2(t)$ when the input is $x_2(t) = 2u(t)$.

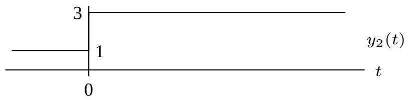
(c) Is the system homogeneous?

(d) Is the system linear?

(a) If the input is $x_1(t) = u(t)$ then the output is $y_1(t) = x_1(t) + 1 = u(t) + 1$:



(b) If the input is $x_2(t) = 2u(t)$ then the output is $y_2(t) = x_2(t) + 1 = 2u(t) + 1$:



(c) For the above cases we have that $x_2(t) = 2x_1(t)$, but we observe that $y_2(t) \neq 2y_1(t)$. Therefore the system is not homogeneous.

(d) Since the system is not homogeneous it is not linear.