EEE2035F: Signals and Systems I

Class Test 1

12 March 2010

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

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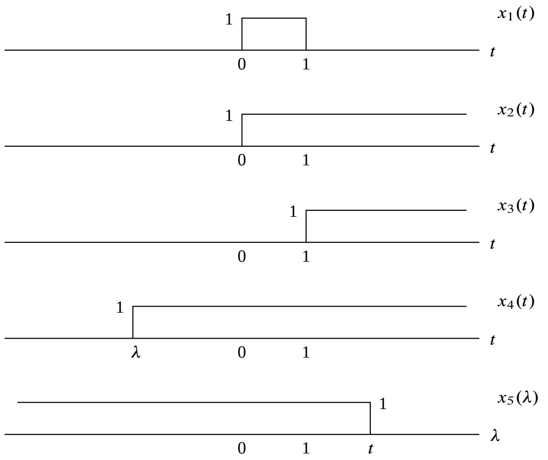
1. (5 marks) Sketch the following signals, where $u(t)$ is the unit step function:
   (a) $x_1(t) = u(t) - u(t-1)$
   (b) $x_2(t) = u(2t)$
   (c) $x_3(t) = u(2(t-1))$
   (d) $x_4(t) = u(t-\lambda)$
   (e) $x_5(\lambda) = u(t-\lambda)$.

   Plots are as follows:



2. (5 marks) A system is defined by the relationship $y(t) = x(-t)$, where $x(t)$ is the input and $y(t)$ the output.
   (a) Is the system causal? Why?
   (b) Is the system linear? Why?
   (c) Is the system time invariant? Why?

(a) The output at time $t = -2$ is given by $y(-2) = x(2)$, which depends on the input at time $t = 2$. This is in the future so the system is not causal.

(b) Suppose we have two input-output pairs $x_1(t) \longrightarrow y_1(t)$ and $x_2(t) \longrightarrow y_2(t)$. Then $y_1(t) = x_1(-t)$ and $y_2(t) = x_2(-t)$. The output for $x(t) = ax_1(t) + bx_2(t)$ is

$$y(t) = x(-t) = ax_1(-t) + bx_2(-t) = ay_1(t) + by_2(t).$$

Since this holds for all input pairs and all values of a and b the system is linear.

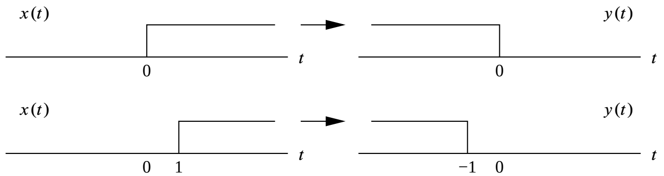
(c) If $x(t) = u(t)$ then the output is $y(t) = u(-t)$, so the following is a valid input-output pair:

$$u(t) \longrightarrow u(-t).$$

If $x(t) = u(t-1)$, then the output is $y(t) = x(-t) = u(-t-1)$, so another valid input-output pair is

$$u(t-1) \longrightarrow u(-t-1) = u(-(t+1)).$$

Sketching these pairs:



Evidently delaying the input by one time unit does not result in the output being delayed by one time unit, so the system is not time invariant.

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3. (5 marks) Suppose $x(t)$ is the signal $x(t) = e^{-2t}u(t)$. Find the following signals, giving a precise mathematical expression for the answer in each case:
   (a) $y_1(t) = \frac{d}{dt}x(t)$
   (b) $y_2(t) = \int_{-\infty}^{t} x(\lambda)d\lambda$.

Drawing the signals is best, because it gives insight, but algebraic solutions are presented here.

(a) The signal is zero for $t < 0$, so $x'(t) = 0$ for $t < 0$. For $t > 0$ the signal is $x(t) = e^{-2t}$, so the derivative is $x'(t) = -2e^{-2t}$ over this range. There is a discontinuity of size 1 at the origin, which introduces an additional term $\delta(t)$ in the solution. The final (generalised) derivative can therefore be written as

$$y_1(t) = \delta(t) - 2e^{-2t}u(t).$$

(b) The signal $x(t)$ is zero for $t < 0$, so $y_2(t)$ is zero over this range (area to the left of $x(\tau)$ for $t < 0$. For $t > 0$ we need to evaluate the integral

$$\begin{aligned} y_2(t) &= \int_{-\infty}^{t} x(\tau)d\tau = \int_{-\infty}^{t} e^{-2\tau}u(\tau)d\tau = \int_{0}^{t} e^{-2\tau}d\tau \\ &= \left[\frac{1}{-2}e^{-2\tau}\right]_{t=0}^{t} = \frac{1}{-2}e^{-2t} - \frac{1}{-2} = \frac{1}{2}(1 - e^{-2t}). \end{aligned}$$

An expression for the overall solution is therefore $y_2(t) = \frac{1}{2}(1 - e^{-2t})u(t)$.

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4. (5 marks) Find and plot $y(t) = u(t) * u(t)$.

Graphical methods are appropriate (and generally preferable), or one could evaluate the convolution integral algebraically:

$$y(t) = u(t) * u(t) = \int_{-\infty}^{\infty} u(\tau)u(t-\tau)d\tau.$$

We are integrating over τ. If $t < 0$ then the product $u(\tau)u(t-\tau)$ is always zero, so $y(t) = 0$. For $t \geq 0$, $u(\tau) = 0$ for $\tau < 0$ so we can make the lower integration limit zero, and $u(t-\tau)$ is zero for $\tau > t$, so the upper limit can be made t. Thus for $t \geq 0$ we have

$$y(t) = \int_0^t u(\tau)u(t-\tau)d\tau.$$

The integrand is exactly one over these integration limits, so

$$y(t) = \int_0^t 1 d\tau = [\tau]_{\tau=0}^{\tau=t} = t.$$

Thus $y(t) = 0$ for $t < 0$ and $y(t) = t$ for $t \geq 0$, and the final expression for the output can be written as

$$y(t) = tu(t).$$

Note that the graphical method takes care of the two different cases as a matter of course, and is therefore easier to work with. A plot of the output is as follows: