

EEE2035F: Signals and Systems I

Class Test 1

12 March 2010

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
 - This test has *four* questions, totalling 20 marks.
 - Answer *all* the questions.
 - You have 45 minutes.
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1. (5 marks) Sketch the following signals, where $u(t)$ is the unit step function:

(a) $x_1(t) = u(t) - u(t - 1)$

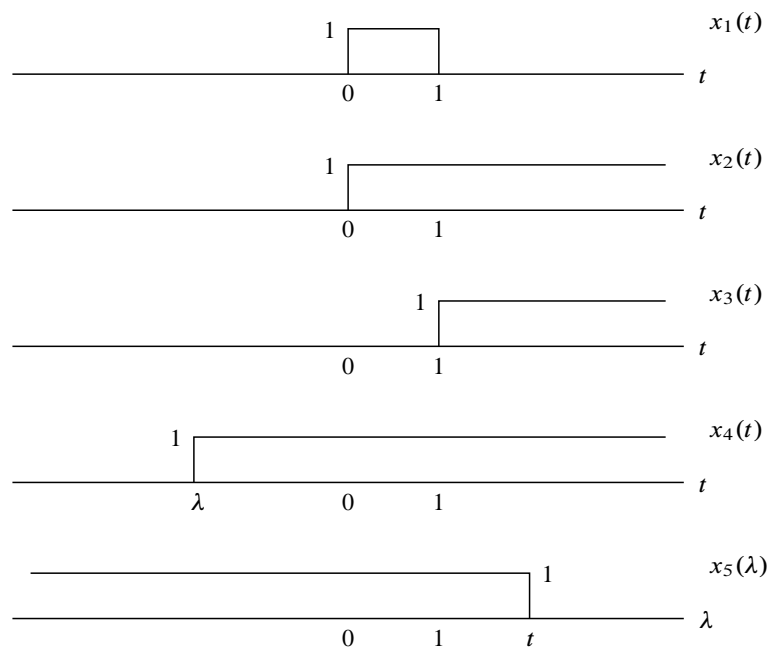
(b) $x_2(t) = u(2t)$

(c) $x_3(t) = u(2(t - 1))$

(d) $x_4(t) = u(t - \lambda)$

(e) $x_5(\lambda) = u(t - \lambda)$.

Plots are as follows:



2. (5 marks) A system is defined by the relationship $y(t) = x(-t)$, where $x(t)$ is the input and $y(t)$ the output.

- (a) Is the system causal? Why?
- (b) Is the system linear? Why?
- (c) Is the system time invariant? Why?

- (a) The output at time $t = -2$ is given by $y(-2) = x(2)$, which depends on the input at time $t = 2$. This is in the future so the system is not causal.
- (b) Suppose we have two input-output pairs $x_1(t) \rightarrow y_1(t)$ and $x_2(t) \rightarrow y_2(t)$. Then $y_1(t) = x_1(-t)$ and $y_2(t) = x_2(-t)$. The output for $x(t) = ax_1(t) + bx_2(t)$ is

$$y(t) = x(-t) = ax_1(-t) + bx_2(-t) = ay_1(t) + by_2(t).$$

Since this holds for all input pairs and all values of a and b the system is linear.

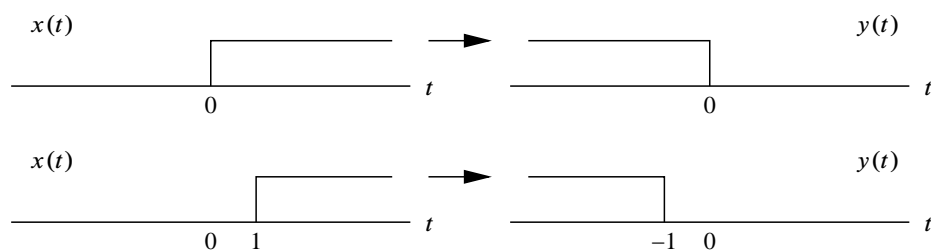
- (c) If $x(t) = u(t)$ then the output is $y(t) = u(-t)$, so the following is a valid input-output pair:

$$u(t) \rightarrow u(-t).$$

If $x(t) = u(t - 1)$, then the output is $y(t) = x(-t) = u(-t - 1)$, so another valid input-output pair is

$$u(t - 1) \rightarrow u(-t - 1) = u(-(t + 1)).$$

Sketching these pairs:



Evidently delaying the input by one time unit does not result in the output being delayed by one time unit, so the system is not time invariant.

3. (5 marks) Suppose $x(t)$ is the signal $x(t) = e^{-2t}u(t)$. Find the following signals, giving a precise mathematical expression for the answer in each case:

(a) $y_1(t) = \frac{d}{dt}x(t)$

(b) $y_2(t) = \int_{-\infty}^t x(\lambda)d\lambda.$

Drawing the signals is best, because it gives insight, but algebraic solutions are presented here.

(a) The signal is zero for $t < 0$, so $x'(t) = 0$ for $t < 0$. For $t > 0$ the signal is $x(t) = e^{-2t}$, so the derivative is $x'(t) = -2e^{-2t}$ over this range. There is a discontinuity of size 1 at the origin, which introduces an additional term $\delta(t)$ in the solution. The final (generalised) derivative can therefore be written as

$$y_1(t) = \delta(t) - 2e^{-2t}u(t).$$

(b) The signal $x(t)$ is zero for $t < 0$, so $y_2(t)$ is zero over this range (area to the left of $x(\tau)$ for $t < 0$. For $t > 0$ we need to evaluate the integral

$$\begin{aligned} y_2(t) &= \int_{-\infty}^t x(\tau)d\tau = \int_{-\infty}^t e^{-2\tau}u(\tau)d\tau = \int_0^t e^{-2\tau}d\tau \\ &= \left[\frac{1}{-2}e^{-2\tau} \right]_{\tau=0}^t = \frac{1}{-2}e^{-2t} - \frac{1}{-2} = \frac{1}{2}(1 - e^{-2t}). \end{aligned}$$

An expression for the overall solution is therefore $y_2(t) = \frac{1}{2}(1 - e^{-2t})u(t)$.

4. (5 marks) Find and plot $y(t) = u(t) * u(t)$.

Graphical methods are appropriate (and generally preferable), or one could evaluate the convolution integral algebraically:

$$y(t) = u(t) * u(t) = \int_{-\infty}^{\infty} u(\tau)u(t - \tau)d\tau.$$

We are integrating over τ . If $t < 0$ then the product $u(\tau)u(t - \tau)$ is always zero, so $y(t) = 0$. For $t \geq 0$, $u(\tau) = 0$ for $\tau < 0$ so we can make the lower integration limit zero, and $u(t - \tau)$ is zero for $\tau > t$, so the upper limit can be made t . Thus for $t \geq 0$ we have

$$y(t) = \int_0^t u(\tau)u(t - \tau)d\tau.$$

The integrand is exactly one over these integration limits, so

$$y(t) = \int_0^t 1d\tau = [\tau]_{\tau=0}^{\tau=t} = t.$$

Thus $y(t) = 0$ for $t < 0$ and $y(t) = t$ for $t \geq 0$, and the final expression for the output can be written as

$$y(t) = tu(t).$$

Note that the graphical method takes care of the two different cases as a matter of course, and is therefore easier to work with. A plot of the output is as follows:

