

EEE2035F: Signals and Systems I

Class Test 1

20 March 2009

SOLUTIONS

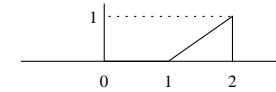
Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

1. (6 marks) Suppose $x(t)$ is the signal below:



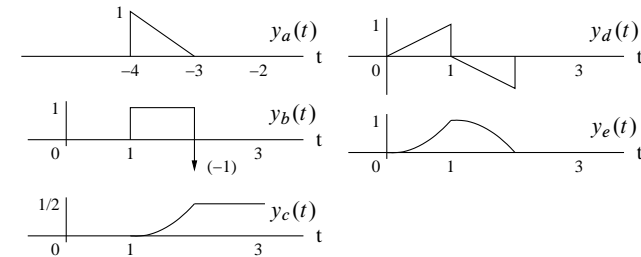
Sketch the following:

- (a) $y_a(t) = x(-2 - t)$
 (b) $y_b(t) = \frac{d}{dt}x(t)$ (generalised derivative)
 (c) $y_c(t) = \int_{-\infty}^t x(\tau) d\tau$
 (d) $y_d(t) = [\delta(t + 1) - \delta(t)] * x(t)$
 (e) $y_e(t) = [u(t + 1) - u(t)] * x(t)$ (Hint: use the result from part (d)).

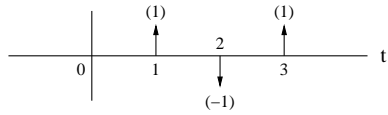
- (a) Nothing to work out
 (b) $y_b(t) = p_1(t - 3/2) - \delta(t - 2)$
 (c) $y_c(t) = 0$ for $t < 1$, $y_c(t) = 1/2$ for $t \geq 2$. For $1 \leq t < 2$,

$$y_c(t) = \int_1^t (\lambda - 1) d\lambda = [1/2\lambda^2 - \lambda]_{\lambda=1}^t = 1/2(t - 1)^2$$

- (d) $y_d(t) = \delta(t + 1) * x(t) - \delta(t) * x(t) = x(t + 1) - x(t)$
 (e) Since $\dot{y}_e(t) = [\dot{u}(t + 1) - \dot{u}(t)] * x(t) = [\delta(t + 1) - \delta(t)] * x(t) = y_d(t)$, we just need to use indefinite integration to find the result: $y_e(t) = \int_{-\infty}^t y_d(\tau) d\tau$.



2. (4 marks) Suppose a LTI system has the impulse response $h(t)$ shown below:



Find the output of the system for each of the following input signals:

- (a) $x_a(t) = \delta(t)$
- (b) $x_b(t) = \delta(t - 1)$
- (c) $x_c(t) = u(t)$.

It may be convenient to write $h(t) = \delta(t - 1) - \delta(t - 2) + \delta(t - 3)$, so

- (a) $y_a(t) = h(t) * x_a(t) = h(t) * \delta(t) = h(t) = \delta(t - 1) - \delta(t - 2) + \delta(t - 3)$
- (b) $y_b(t) = h(t) * x_b(t) = h(t) * \delta(t - 1) = h(t - 1) = \delta(t - 2) - \delta(t - 3) + \delta(t - 4)$
- (c) $y_c(t) = h(t) * x_c(t) = [\delta(t - 1) - \delta(t - 2) + \delta(t - 3)] * u(t)$, so
 $y_c(t) = u(t - 1) - u(t - 2) + u(t - 3)$.

3. (5 marks) The input $x(t)$ and output $y(t)$ of a given system satisfies the following relationship for all t :

$$y(t + 4) = \int_t^{t+1} x(\lambda) d\lambda.$$

- (a) Is the system causal? Explain.
- (b) What is the output of the system when the input $x(t)$ is the unit step?

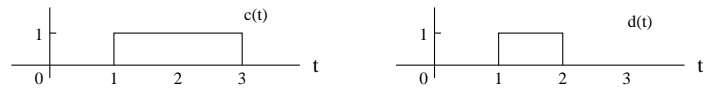
True for all t , so we can replace t with $t - 4$ in the relationship:

$$y(t) = \int_{t-4}^{t-3} x(\lambda) d\lambda.$$

- (a) To find the output at $t = t_0$ we need to integrate the input over the range $t_0 - 4$ to $t_0 - 3$, so we need to know $x(t)$ for $t_0 - 4 \leq x(t) \leq t_0 - 3$. These values are all in the past, so the system is causal.
- (b) For $x(t) = u(t)$, $y(t) = \int_{t-4}^{t-3} u(\lambda) d\lambda$. As long as the upper limit of the integral is negative, no nonzero part of the unit step is being integrated: thus $y(t) = 0$ for $t - 3 < 0$. When the lower limit of the integral is greater than zero, one unit of area is being integrated over the positive part of the unit step: thus $y(t) = 1$ for $t - 4 > 0$. For $3 < t < 4$, have $y(t) = \int_0^{t-3} d\lambda = t$. Thus



4. (5 marks) Use the graphical method to convolve the two signals below:



The convolution $y(t) = c(t) * d(t)$ is as follows:

