EEE2035F: Signals and Systems I

Class Test 1

20 March 2009

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

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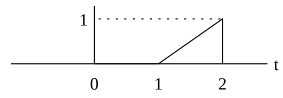
1. (6 marks) Suppose $x(t)$ is the signal below:



Sketch the following:

(a) $y_a(t) = x(-2 - t)$

(b) $y_b(t) = \frac{d}{dt}x(t)$ (generalised derivative)

(c) $y_c(t) = \int_{-\infty}^{t} x(\tau)d\tau$

(d) $y_d(t) = [\delta(t + 1) - \delta(t)] * x(t)$

(e) $y_e(t) = [u(t + 1) - u(t)] * x(t)$ (Hint: use the result from part (d)).

(a) Nothing to work out

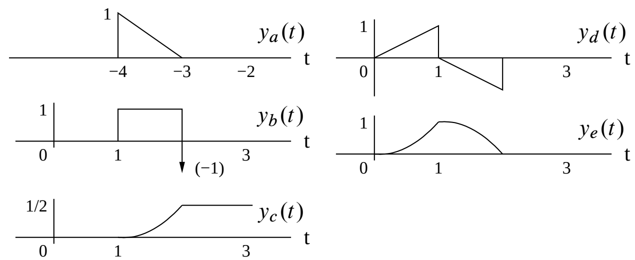
(b) $y_b(t) = p_1(t - 3/2) - \delta(t - 2)$

(c) $y_c(t) = 0$ for $t < 1$, $y_c(t) = 1/2$ for $t \geq 2$. For $1 \leq t < 2$,

$$y_c(t) = \int_1^t (\lambda - 1)d\lambda = [1/2\lambda^2 - \lambda]_{\lambda=1}^t = 1/2(t - 1)^2$$

(d) $y_d(t) = \delta(t + 1) * x(t) - \delta(t) * x(t) = x(t + 1) - x(t)$

(e) Since $\dot{y}_e(t) = [\dot{u}(t + 1) - \dot{u}(t)] * x(t) = [\delta(t + 1) - \delta(t)] * x(t) = y_d(t)$, we just neeed to use indefinite integration to find the result: $y_e(t) = \int_{-\infty}^{t} y_d(\tau)d\tau$.



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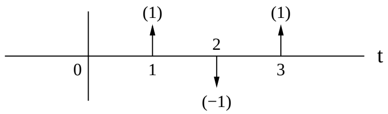
2. (4 marks) Suppose a LTI system has the impulse response $h(t)$ shown below:



Find the output of the system for each of the following input signals:

(a) $x_a(t) = \delta(t)$

(b) $x_b(t) = \delta(t-1)$

(c) $x_c(t) = u(t)$.

It may be convenient to write $h(t) = \delta(t-1) - \delta(t-2) + \delta(t-3)$, so

(a) $y_a(t) = h(t) * x_a(t) = h(t) * \delta(t) = h(t) = \delta(t-1) - \delta(t-2) + \delta(t-3)$

(b) $y_b(t) = h(t) * x_b(t) = h(t) * \delta(t-1) = h(t-1) = \delta(t-2) - \delta(t-3) + \delta(t-4)$

(c) $y_c(t) = h(t) * x_c(t) = [\delta(t-1) - \delta(t-2) + \delta(t-3)] * u(t)$, so
$y_c(t) = u(t-1) - u(t-2) + u(t-3)$.

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3. (5 marks) The input $x(t)$ and output $y(t)$ of a given system satisfies the following relationship for all t:

$$y(t+4) = \int_{t}^{t+1} x(\lambda) d\lambda.$$

(a) Is the system causal? Explain.

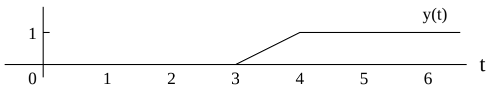
(b) What is the output of the system when the input $x(t)$ is the unit step?

True for all t, so we can replace t with $t-4$ in the relationship:

$$y(t) = \int_{t-4}^{t-3} x(\lambda) d\lambda.$$

(a) To find the output at $t = t_0$ we need to integrate the input over the range $t_0 - 4$ to $t_0 - 3$, so we need to know $x(t)$ for $t_0 - 4 \leq x(t) \leq t_0 - 3$. These values are all in the past, so the system is causal.

(b) For $x(t) = u(t)$, $y(t) = \int_{t-4}^{t-3} u(\lambda) d\lambda$. As long as the upper limit of the integral is negative, no nonzero part of the unit step is being integrated: thus $y(t) = 0$ for $t - 3 < 0$. When the lower limit of the integral is greater than zero, one unit of area is being integrated over the positive part of the unit step: thus $y(t) = 1$ for $t - 4 > 0$. For $3 < t < 4$, have $y(t) = \int_0^t d\lambda = t$. Thus



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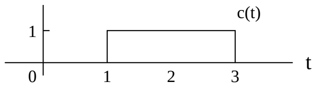
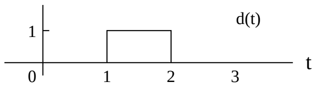
4. (5 marks) Use the graphical method to convolve the two signals below:





The convolution $y(t) = c(t) * d(t)$ is as follows: