

# EEE2035F: Signals and Systems I

## Class Test 1

3 April 2008

Name:

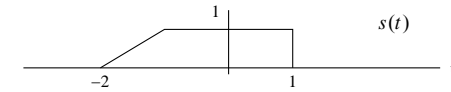
Student number:

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### Information

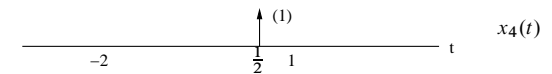
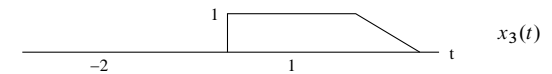
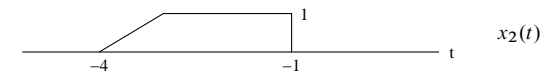
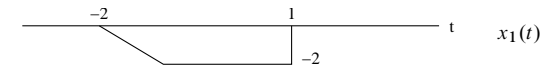
- The test is closed-book.
  - This test has *four* questions, totalling 20 marks.
  - Answer *all* the questions.
  - You have 45 minutes.
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1. (5 marks) Consider the signal  $s(t)$  below:



Plot the following:

- (a)  $x_1(t) = -2s(t)$
- (b)  $x_2(t) = s(t + 2)$
- (c)  $x_3(t) = s(1 - t)$
- (d)  $x_4(t) = s(t)\delta(\frac{1}{2} - t)$ .



2. (5 marks) A system has an impulse response

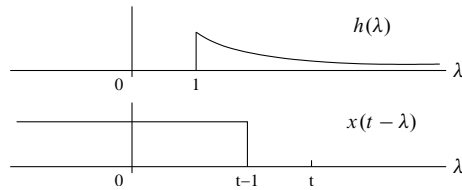
$$h(t) = 3e^{-10t}u(t-1)$$

Find the response of the system to the input  $x(t) = u(t-1)$ .

The required quantity is

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda,$$

which is comprised of the following signals:



When  $t-1 < 1$  there is no overlap so the output is zero. Thus  $y(t) = 0$  for  $t < 2$ . For  $t \geq 2$  the output is

$$\begin{aligned} y(t) &= \int_1^{t-1} h(\lambda)x(t-\lambda)d\lambda = \int_1^{t-1} h(\lambda)d\lambda = \int_1^{t-1} 3e^{-10\lambda}d\lambda \\ &= 3 \left[ -1/10e^{-10\lambda} \right]_{\lambda=1}^{\lambda=t-1} = \frac{3}{10}(e^{-10} - e^{-10(t-1)}). \end{aligned}$$

It's probably easier to use the derivative property to find

$$\dot{y}(t) = h(t) * \dot{x}(t) = h(t) * \delta(t-1) = h(t-1) \text{ and then just integrate this to find } y(t).$$


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3. (5 marks) If  $h(t) = e^{-2t}u(t)$  and  $x(t) = \delta(t-1) + 2\delta(t+2)$ , find the signal defined by

$$y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda.$$

We're being asked to do the convolution  $y(t) = x(t) * h(t) = h(t) * x(t)$ . Considering  $h(t)$  to be the impulse response of a system (with  $x(t)$  the input) we know the response to an impulse at the origin is  $h(t)$ :

$$\delta(t) \implies e^{-2t}u(t).$$

Through linearity and time invariance, the following input/output pairs must also be valid:

$$\delta(t-1) \implies e^{-2(t-1)}u(t-1) \quad (\text{time invariance})$$

$$\delta(t+2) \implies e^{-2(t+2)}u(t+2) \quad (\text{time invariance})$$

$$2\delta(t+2) \implies 2e^{-2(t+2)}u(t+2) \quad (\text{homogeneity})$$

$$\delta(t-1) + 2\delta(t+2) \implies e^{-2(t-1)}u(t-1) + 2e^{-2(t+2)}u(t+2) \quad (\text{additivity}).$$

The output for the given  $x(t)$  is therefore

$$y(t) = e^{-2(t-1)}u(t-1) + 2e^{-2(t+2)}u(t+2).$$


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4. (5 marks) If  $h(t) = e^{2t}$  and  $x(t) = \delta(t - 1) + 2\delta(t + 2)$ , find the signal defined by

$$y(t) = \int_{-\infty}^t x(\lambda)h(\lambda)d\lambda.$$

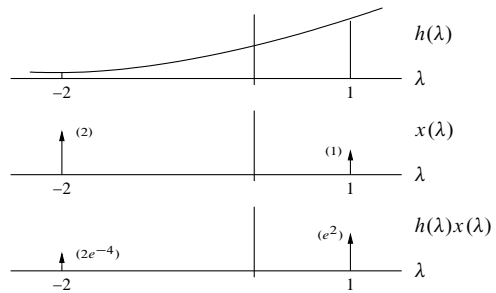
We have

$$\begin{aligned} y(t) &= \int_{-\infty}^t (\delta(\lambda - 1) + 2\delta(\lambda + 2))e^{2\lambda}d\lambda \\ &= \int_{-\infty}^t \delta(\lambda - 1)e^{2\lambda}d\lambda + 2 \int_{-\infty}^t \delta(\lambda + 2)e^{2\lambda}d\lambda \\ &= \int_{-\infty}^t \delta(\lambda - 1)e^{2(1)}d\lambda + 2 \int_{-\infty}^t \delta(\lambda + 2)e^{2(-2)}d\lambda \quad (\text{sifting property}) \\ &= e^2 \int_{-\infty}^t \delta(\lambda - 1)d\lambda + 2e^{-4} \int_{-\infty}^t \delta(\lambda + 2)d\lambda. \end{aligned}$$

Since  $\int_{-\infty}^t \delta(\lambda - 1)d\lambda = u(t - 1)$  and  $\int_{-\infty}^t \delta(\lambda + 2)d\lambda = u(t + 2)$ , we have

$$y(t) = e^2u(t - 1) + 2e^{-4}u(t + 2).$$

Else plot the integrand as a function of  $\lambda$ :



The indefinite integral of this last signal is the required result.

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