EEE2035F: Signals and Systems I

Class Test 1

3 April 2008

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

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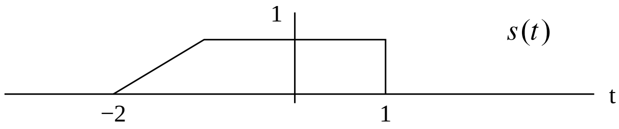
1. (5 marks) Consider the signal $s(t)$ below:



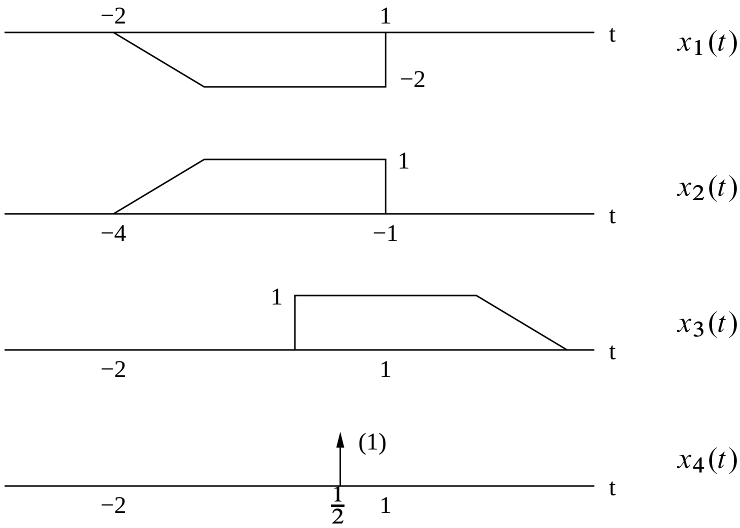
Plot the following:

(a) $x_1(t) = -2s(t)$

(b) $x_2(t) = s(t+2)$

(c) $x_3(t) = s(1-t)$

(d) $x_4(t) = s(t)\delta(\frac{1}{2} - t)$.



2. (5 marks) A system has an impulse response

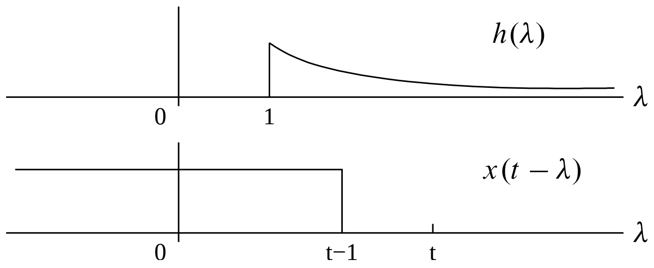
$$h(t) = 3e^{-10t}u(t-1)$$

Find the response of the system to the input $x(t) = u(t-1)$.

The required quantity is

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda,$$

which is comprised of the following signals:



When $t-1 < 1$ there is no overlap so the output is zero. Thus $y(t) = 0$ for $t < 2$. For $t \geq 2$ the output is

$$y(t) = \int_{1}^{t-1} h(\lambda)x(t-\lambda)d\lambda = \int_{1}^{t-1} h(\lambda)d\lambda = \int_{1}^{t-1} 3e^{-10\lambda}d\lambda$$
$$= 3\left[-1/10e^{-10\lambda}\right]_{\lambda=1}^{\lambda=t-1} = \frac{3}{10}(e^{-10} - e^{-10(t-1)}).$$

It's probably easier to use the derivative property to find $\dot{y}(t) = h(t) * \dot{x}(t) = h(t) * \delta(t-1) = h(t-1)$ and then just integrate this to find $y(t)$.

3. (5 marks) If $h(t) = e^{-2t}u(t)$ and $x(t) = \delta(t-1) + 2\delta(t+2)$, find the signal defined by

$$y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda.$$

We're being asked to do the convolution $y(t) = x(t) * h(t) = h(t) * x(t)$. Considering $h(t)$ to be the impulse response of a system (with $x(t)$ the input) we know the response to an impulse at the origin is $h(t)$:

$$\delta(t) \Longrightarrow e^{-2t}u(t).$$

Through linearity and time invariance, the following input/output pairs must also be valid:

$$\begin{aligned} \delta(t-1) &\Longrightarrow e^{-2(t-1)}u(t-1) && \text{(time invariance)} \\ \delta(t+2) &\Longrightarrow e^{-2(t+2)}u(t+2) && \text{(time invariance)} \\ 2\delta(t+2) &\Longrightarrow 2e^{-2(t+2)}u(t+2) && \text{(homogeneity)} \\ \delta(t-1) + 2\delta(t+2) &\Longrightarrow e^{-2(t-1)}u(t-1) + 2e^{-2(t+2)}u(t+2) && \text{(additivity).} \end{aligned}$$

The output for the given $x(t)$ is therefore

$$y(t) = e^{-2(t-1)}u(t-1) + 2e^{-2(t+2)}u(t+2).$$

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4. (5 marks) If $h(t) = e^{2t}$ and $x(t) = \delta(t-1) + 2\delta(t+2)$, find the signal defined by

$$y(t) = \int_{-\infty}^{t} x(\lambda)h(\lambda)d\lambda.$$

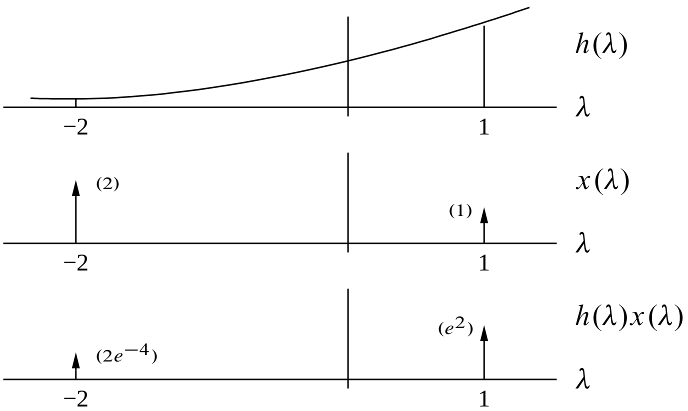
We have

$$\begin{aligned}
y(t) &= \int_{-\infty}^{t} (\delta(\lambda-1) + 2\delta(\lambda+2))e^{2\lambda}d\lambda \\
&= \int_{-\infty}^{t} \delta(\lambda-1)e^{2\lambda}d\lambda + 2\int_{-\infty}^{t} \delta(\lambda+2)e^{2\lambda}d\lambda \\
&= \int_{\infty}^{t} \delta(\lambda-1)e^{2(1)}d\lambda + 2\int_{-\infty}^{t} \delta(\lambda+2)e^{2(-2)}d\lambda \quad \text{(sifting property)} \\
&= e^{2}\int_{\infty}^{t} \delta(\lambda-1)d\lambda + 2e^{-4}\int_{-\infty}^{t} \delta(\lambda+2)d\lambda.
\end{aligned}$$

Since $\int_{-\infty}^{t} \delta(\lambda-1)d\lambda = u(t-1)$ and $\int_{-\infty}^{t} \delta(\lambda+2)d\lambda = u(t+2)$, we have

$$y(t) = e^{2}u(t-1) + 2e^{-4}u(t+2).$$

Else plot the integrand as a function of λ:



The indefinite integral of this last signal is the required result.
