# EEE2035F: Signals and Systems I

## Class Test 1

22 March 2007

**Name:**

**Student number:**

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### Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- There is an information sheet attached at the end of this paper.
- Answer *all* the questions.
- You have 45 minutes.

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1. (10 marks) Given

$$x(t) = \begin{cases} t & 0 \leq t < 1 \\ 1 & 1 \leq t < 2 \\ 0 & \text{otherwise,} \end{cases}$$

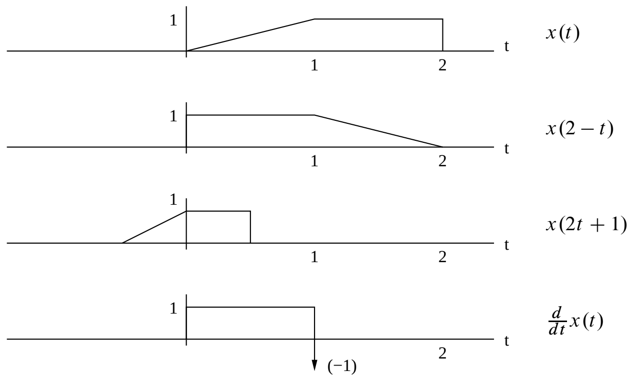
sketch

(a) $x(t)$

(b) $y_1(t) = x(2t+1)$

(c) $y_2(t) = x(2-t)$.

(d) the generalised derivative $y_3(t) = \frac{dx(t)}{dt}$.

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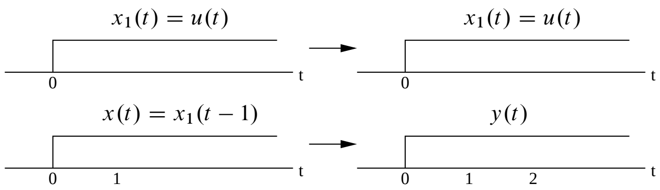
2. (10 marks) Show by counterexample that the system with input-output relation $y(t) = x(t/2)$ is not time invariant.

If the system is time invariant and $y_1(t)$ is the response to input $x_1(t)$, then the response to $x_1(t-c)$ must be $y_1(t-c)$ for all $c$.

Just about any signal works as a counterexample:




Is $y(t) = y_1(t-1)$? No, so system is not time invariant.

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3. (10 marks) Suppose $x(t)$ in question 1 is the input to an LTI system with impulse response $h(t)$. Find the output for the two cases of:

(a) $h(t) = \delta(t)$

(b) $h(t) = \delta(t-2)$.

(a) The output is $y(t) = h(t) * x(t) = \delta(t) * x(t) = x(t)$ (since $\delta(t)$ is the identity element of convolution).

(b) The output is

$$\begin{aligned} y(t) = h(t) * x(t) &= \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda \\ &= \int_{-\infty}^{\infty} \delta(\lambda-2)x(t-\lambda)d\lambda = \int_{-\infty}^{\infty} \delta(\lambda-2)x(t-2)d\lambda \\ &= x(t-2)\int_{-\infty}^{\infty} \delta(\lambda-2)d\lambda = x(t-2). \end{aligned}$$

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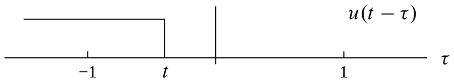
4. (10 marks) Find $y(t) = p_2(t) * u(t)$ using graphical or algebraic methods.
(In case you forget, the signal $p_2(t)$ is a pulse of height 1, centered on the origin and of total width 2, and $u(t)$ is the unit step signal.)

The convolution output is

$$y(t) = \int_{-\infty}^{\infty} p_2(\tau)u(t-\tau)d\tau = \int_{-1}^{1} p_2(\tau)u(t-\tau)d\tau$$
$$= \int_{-1}^{1} (1)u(t-\tau)d\tau = \int_{-1}^{1} u(t-\tau)d\tau.$$

Now $u(t-\tau)$ is the signal below:




There are three cases:

- For $t < -1$ we have $y(t) = \int_{-1}^{1}(0)d\tau = 0$.
- For $-1 \le t \le 1$ we have $y(t) = \int_{-1}^{t}(1)d\tau = t + 1$.
- For $t > 1$ we have $y(t) = \int_{-1}^{1}(1)d\tau = 2$.

# INFORMATION SHEET

## Fourier transform properties

| Property | Transform Pair/Property |
|---|---|
| Linearity | $ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$ |
| Time shift | $x(t-c) \leftrightarrow X(\omega)e^{-j\omega c}$ |
| Time scaling | $x(at) \leftrightarrow \frac{1}{a}X(\frac{\omega}{a}) \quad a > 0$ |
| Time reversal | $x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$ |
| Multiplication by power of $t$ | $t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n}X(\omega) \quad n = 1, 2, \ldots$ |
| Multiplication by complex exponential | $x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0$ real |
| Multiplication by $\cos(\omega_0 t)$ | $x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$ |
| Differentiation in time domain | $\frac{d^n}{dt^n}x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \ldots$ |
| Integration | $\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$ |
| Convolution in time domain | $x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$ |
| Multiplication in time domain | $x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$ |
| Parseval's theorem | $\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}\overline{X(\omega)}V(\omega)d\omega$ |
| Parseval's theorem (special case) | $\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}\|X(\omega)\|^2 d\omega$ |
| Duality | $X(t) \leftrightarrow 2\pi x(-\omega)$ |

## Common Fourier Transform Pairs

| $x(t)$ | $X(\omega)$ |
|---|---|
| $1 \quad (-\infty < t < \infty)$ | $2\pi\delta(\omega)$ |
| $-0.5 + u(t)$ | $\frac{1}{j\omega}$ |
| $u(t)$ | $\pi\delta(\omega) + \frac{1}{j\omega}$ |
| $\delta(t)$ | $1$ |
| $\delta(t-c)$ | $e^{-j\omega c}$ ($c$ any real number) |
| $e^{-bt}u(t)$ | $\frac{1}{j\omega + b} \quad (b > 0)$ |
| $e^{j\omega_0 t}$ | $2\pi\delta(\omega - \omega_0)$ ($\omega_0$ any real number) |
| $p_\tau(t)$ | $\tau \text{sinc} \frac{\tau\omega}{2\pi}$ |
| $\tau \text{sinc} \frac{\tau t}{2\pi}$ | $2\pi p_\tau(\omega)$ |
| $\left(1 - \frac{2\|t\|}{\tau}\right) p_\tau(t)$ | $\frac{\tau}{2}\text{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$ |
| $\frac{\tau}{2}\text{sinc}^2 \frac{\tau t}{4\pi}$ | $2\pi\left(1 - \frac{2\|\omega\|}{\tau}\right) p_\tau(\omega)$ |
| $\cos(\omega_0 t + \theta)$ | $\pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$ |
| $\sin(\omega_0 t + \theta)$ | $j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$ |

## Trigonometric identities

$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) \qquad \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$