

EEE2035F: Signals and Systems I

Class Test 1

22 March 2007

Name:

Student number:

Information

- The test is closed-book.
 - This test has *four* questions, totalling 20 marks.
 - There is an information sheet attached at the end of this paper.
 - Answer *all* the questions.
 - You have 45 minutes.
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1. (10 marks) Given

$$x(t) = \begin{cases} t & 0 \leq t < 1 \\ 1 & 1 \leq t < 2 \\ 0 & \text{otherwise,} \end{cases}$$

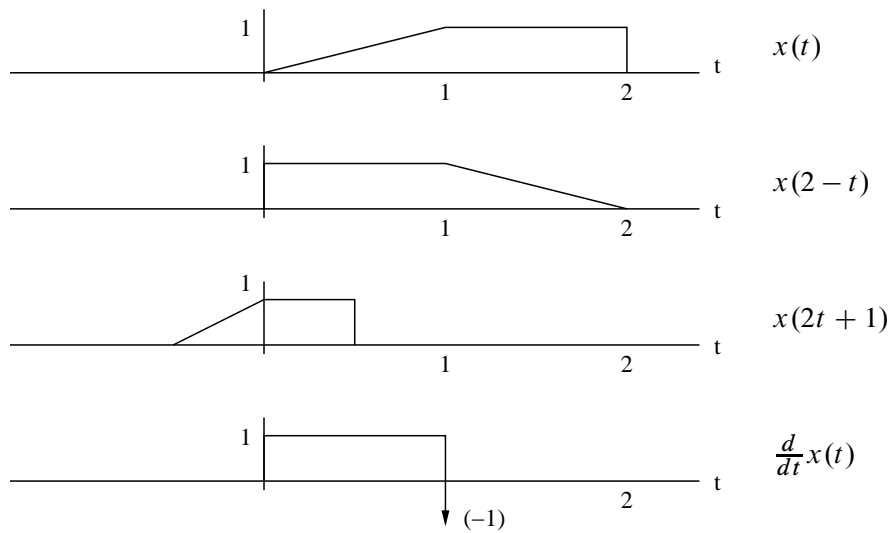
sketch

(a) $x(t)$

(b) $y_1(t) = x(2t + 1)$

(c) $y_2(t) = x(2 - t)$.

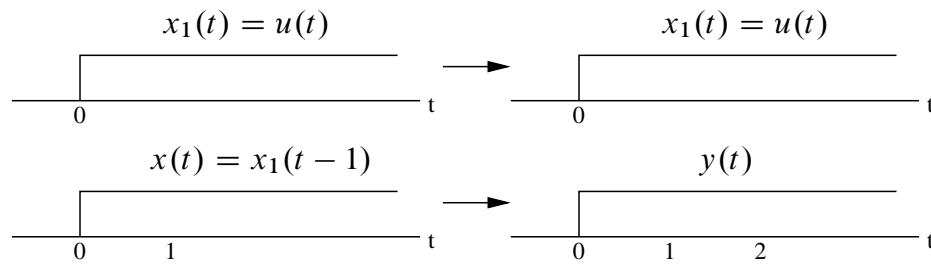
(d) the generalised derivative $y_3(t) = \frac{dx(t)}{dt}$.



2. (10 marks) Show by counterexample that the system with input-output relation $y(t) = x(t/2)$ is not time invariant.

If the system is time invariant and $y_1(t)$ is the response to input $x_1(t)$, then the response to $x_1(t - c)$ must be $y_1(t - c)$ for all c .

Just about any signal works as a counterexample:



Is $y(t) = y_1(t - 1)$? No, so system is not time invariant.

3. (10 marks) Suppose $x(t)$ in question 1 is the input to an LTI system with impulse response $h(t)$. Find the output for the two cases of:

(a) $h(t) = \delta(t)$

(b) $h(t) = \delta(t - 2)$.

(a) The output is $y(t) = h(t) * x(t) = \delta(t) * x(t) = x(t)$ (since $\delta(t)$ is the identity element of convolution).

(b) The output is

$$\begin{aligned} y(t) &= h(t) * x(t) = \int_{-\infty}^{\infty} h(\lambda)x(t - \lambda)d\lambda \\ &= \int_{-\infty}^{\infty} \delta(\lambda - 2)x(t - \lambda)d\lambda = \int_{-\infty}^{\infty} \delta(\lambda - 2)x(t - 2)d\lambda \\ &= x(t - 2) \int_{-\infty}^{\infty} \delta(\lambda - 2)d\lambda = x(t - 2). \end{aligned}$$

4. (10 marks) Find $y(t) = p_2(t) * u(t)$ using graphical or algebraic methods.
(In case you forget, the signal $p_2(t)$ is a pulse of height 1, centered on the origin and of total width 2, and $u(t)$ is the unit step signal.)

The convolution output is

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} p_2(\tau)u(t - \tau)d\tau = \int_{-1}^1 p_2(\tau)u(t - \tau)d\tau \\ &= \int_{-1}^1 (1)u(t - \tau)d\tau = \int_{-1}^1 u(t - \tau)d\tau. \end{aligned}$$

Now $u(t - \tau)$ is the signal below:



There are three cases:

- For $t < -1$ we have $y(t) = \int_{-1}^1 (0)d\tau = 0$.
 - For $-1 \leq t \leq 1$ we have $y(t) = \int_{-1}^t (1)d\tau = t + 1$.
 - For $t > 1$ we have $y(t) = \int_{-1}^1 (1)d\tau = 2$.
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INFORMATION SHEET

Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t - c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{ a }X\left(\frac{\omega}{a}\right) \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of t	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \dots$
Multiplication by complex exponential	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0 \text{ real}$
Multiplication by $\cos(\omega_0 t)$	$x(t) \cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \dots$
Integration	$\int_{-\infty}^t x(\lambda) d\lambda \leftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$
Convolution in time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi} X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega) d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) ^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

Common Fourier Transform Pairs

$x(t)$	$X(\omega)$
1 $(-\infty < t < \infty)$	$2\pi\delta(\omega)$
$-0.5 + u(t)$	$\frac{1}{j\omega}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t - c)$	$e^{-j\omega c} \quad (c \text{ any real number})$
$e^{-bt}u(t)$	$\frac{1}{j\omega + b} \quad (b > 0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0) \quad (\omega_0 \text{ any real number})$
$p_\tau(t)$	$\tau \text{sinc} \frac{\tau\omega}{2\pi}$
$\tau \text{sinc} \frac{\tau t}{2\pi}$	$2\pi p_\tau(\omega)$
$\left(1 - \frac{2 t }{\tau}\right) p_\tau(t)$	$\frac{\tau}{2} \text{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$
$\frac{\tau}{2} \text{sinc}^2 \frac{\tau t}{4\pi}$	$2\pi \left(1 - \frac{2 \omega }{\tau}\right) p_\tau(\omega)$
$\cos(\omega_0 t + \theta)$	$\pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$
$\sin(\omega_0 t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$

Trigonometric identities

$$\sin(-\theta) = -\sin(\theta) \quad \cos(-\theta) = \cos(\theta) \quad \tan(-\theta) = -\tan(\theta)$$

$$\sin^2(\theta) + \cos^2(\theta) = 1 \quad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$$

$$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) \quad \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$$

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$