

# EEE2035F: Signals and Systems I

## Class Test 1

22 March 2007

Name:

Student number:

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### Information

- The test is closed-book.
  - This test has *four* questions, totalling 20 marks.
  - There is an information sheet attached at the end of this paper.
  - Answer *all* the questions.
  - You have 45 minutes.
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1. (10 marks) Given

$$x(t) = \begin{cases} t & 0 \leq t < 1 \\ 1 & 1 \leq t < 2 \\ 0 & \text{otherwise,} \end{cases}$$

sketch

- (a)  $x(t)$
- (b)  $y_1(t) = x(2t + 1)$
- (c)  $y_2(t) = x(2 - t)$ .
- (d) the generalised derivative  $y_3(t) = \frac{dx(t)}{dt}$ .

2. (10 marks) Show by counterexample that the system with input-output relation  $y(t) = x(t/2)$  is not time invariant.

3. (10 marks) Suppose  $x(t)$  in question 1 is the input to an LTI system with impulse response  $h(t)$ . Find the output for the two cases of:

(a)  $h(t) = \delta(t)$

(b)  $h(t) = \delta(t - 2)$ .

4. (10 marks) Find  $y(t) = p_2(t) * u(t)$  using graphical or algebraic methods.  
(In case you forget, the signal  $p_2(t)$  is a pulse of height 1, centered on the origin and of total width 2, and  $u(t)$  is the unit step signal.)

## INFORMATION SHEET

### Fourier transform properties

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Time shift	$x(t - c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{ a }X(\frac{\omega}{a}) \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by power of $t$	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \dots$
Multiplication by complex exponential	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0 \text{ real}$
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation in time domain	$\frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \dots$
Integration	$\int_{-\infty}^t x(\lambda)d\lambda \leftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$
Convolution in time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi} X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$
Parseval's theorem (special case)	$\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi} \int_{-\infty}^{\infty}  X(\omega) ^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

### Common Fourier Transform Pairs

$x(t)$	$X(\omega)$
1 $(-\infty < t < \infty)$	$2\pi\delta(\omega)$
$-0.5 + u(t)$	$\frac{1}{j\omega}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\delta(t)$	1
$\delta(t - c)$	$e^{-j\omega c} \quad (c \text{ any real number})$
$e^{-bt}u(t)$	$\frac{1}{j\omega + b} \quad (b > 0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0) \quad (\omega_0 \text{ any real number})$
$p_\tau(t)$	$\tau \text{sinc} \frac{\omega\tau}{2}$
$\tau \text{sinc} \frac{\tau t}{2}$	$2\pi p_\tau(\omega)$
$(1 - \frac{2 t }{\tau})p_\tau(t)$	$\frac{\tau}{2} \text{sinc}^2(\frac{\omega\tau}{4\pi})$
$\frac{\tau}{2} \text{sinc}^2 \frac{\tau t}{4\pi}$	$2\pi(1 - \frac{2 \omega\tau }{\tau})p_\tau(\omega)$
$\cos(\omega_0 t + \theta)$	$\pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$
$\sin(\omega_0 t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$

### Trigonometric identities

$$\begin{aligned} \sin(-\theta) &= -\sin(\theta) & \cos(-\theta) &= \cos(\theta) & \tan(-\theta) &= -\tan(\theta) \\ \sin^2(\theta) + \cos^2(\theta) &= 1 & \sin(2\theta) &= 2\sin(\theta)\cos(\theta) \\ \cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta) \\ \sin(\theta_1 + \theta_2) &= \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) & \cos(\theta_1 + \theta_2) &= \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2) \\ e^{j\theta} &= \cos(\theta) + j\sin(\theta) \end{aligned}$$