

## EEE235F Class Test

20 May 2005

Name:

Student number:

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### Information

- The test is closed-book.
  - This test has *three* questions, totalling 30 marks.
  - Answer *all* the questions.
  - You have 45 minutes.
  - There is an information sheet attached to the end of this question paper.
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1. (10 marks) Use time-domain convolution to find the inverse Fourier transform of

$$X(\omega) = \frac{1}{(a + j\omega)^2}.$$

2. (10 marks) A time signal  $x(t)$  has the Fourier transform  $4/(4 + \omega^2)$ . Find the Fourier transforms of the following signals:

(a)  $x_1(t) = x(2t - 3)$

(b)  $x_2(t) = \int_{-\infty}^t x(\tau) d\tau$ .

3. (10 marks) Find the inverse Fourier transform of the signal

$$G(\omega) = \frac{1}{1 + j\omega} \cos(2\omega) e^{-j5\omega}.$$

## INFORMATION SHEET

### Fourier transform properties

| Property                              | Transform Pair/Property  |
|---------------------------------------|--|
| Linearity                             | $ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$  |
| Time shift                            | $x(t - c) \leftrightarrow X(\omega)e^{-j\omega c}$   |
| Time scaling                          | $x(at) \leftrightarrow \frac{1}{ a }X\left(\frac{\omega}{a}\right) \quad a > 0$  |
| Time reversal                         | $x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$  |
| Multiplication by power of $t$        | $t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \dots$   |
| Multiplication by complex exponential | $x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0 \text{ real}$   |
| Differentiation in time domain        | $\frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \dots$  |
| Integration                           | $\int_{-\infty}^t x(\lambda) d\lambda \leftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$                      |
| Convolution in time domain            | $x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$   |
| Multiplication in time domain         | $x(t)v(t) \leftrightarrow \frac{1}{2\pi} X(\omega) * V(\omega)$  |
| Parseval's theorem                    | $\int_{-\infty}^{\infty} x(t)v(t)dt \leftrightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)d\omega$ |
| Duality                               | $X(t) \leftrightarrow 2\pi x(-\omega)$   |

### Common Fourier Transform Pairs

| $x(t)$   | $X(\omega)$   |
|--|---|
| 1 $(-\infty < t < \infty)$                     | $2\pi\delta(\omega)$  |
| $-0.5 + u(t)$                                  | $\frac{1}{j\omega}$   |
| $u(t)$   | $\pi\delta(\omega) + \frac{1}{j\omega}$                                     |
| $\delta(t)$                                    | 1   |
| $\delta(t - c)$                                | $e^{-j\omega c} \quad (c \text{ any real number})$                          |
| $e^{-bt}u(t)$                                  | $\frac{1}{j\omega + b} \quad (b > 0)$                                       |
| $e^{j\omega_0 t}$                              | $2\pi\delta(\omega - \omega_0) \quad (\omega_0 \text{ any real number})$    |
| $p_\tau(t)$                                    | $\tau \operatorname{sinc} \frac{\tau\omega}{2\pi}$                          |
| $\left(1 - \frac{2 t }{\tau}\right) p_\tau(t)$ | $\frac{\tau}{2} \operatorname{sinc}^2 \left(\frac{\tau\omega}{4\pi}\right)$ |