

## Chapter 1 Fundamental Concepts

### Signals

- A **signal** is a **pattern of variation of a physical quantity** as a *function* of time, space, distance, position, temperature, pressure, etc.
- These quantities are usually the **independent variables** of the function defining the signal
- A signal encodes **information**, which is the variation itself

### Signal Processing

- Signal processing is the discipline concerned with **extracting, analyzing, and manipulating the information** carried by signals
- The processing method depends on the type of signal and on the nature of the information carried by the signal

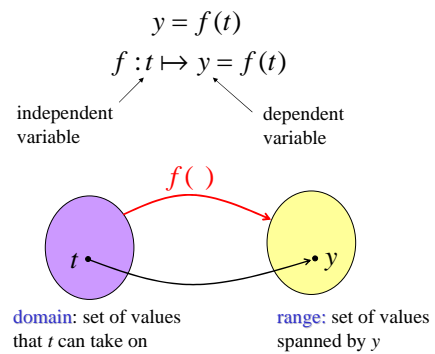
### Characterization and Classification of Signals

- The **type of signal** depends on the nature of the independent variables and on the value of the function defining the signal
- For example, the independent variables can be **continuous or discrete**
- Likewise, the signal can be a **continuous or discrete function** of the independent variables

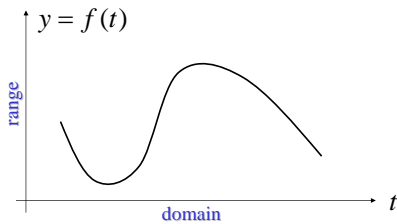
### Characterization and Classification of Signals – Cont'd

- Moreover, the signal can be either a **real-valued function** or a **complex-valued function**
- A signal consisting of a single component is called a **scalar or one-dimensional (1-D) signal**
- A signal consisting of multiple components is called a **vector or multidimensional (M-D) signal**

### Definition of Function from Calculus



## Plot or Graph of a Function



## Continuous-Time (CT) and Discrete-Time (DT) Signals

- A signal  $x(t)$  depending on a continuous temporal variable  $t \in \mathbb{R}$  will be called a **continuous-time (CT) signal**
- A signal  $x[n]$  depending on a discrete temporal variable  $n \in \mathbb{Z}$  will be called a **discrete-time (DT) signal**

## Examples: CT vs. DT Signals

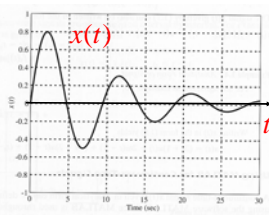


Figure 1.13 MATLAB plot of the signal  $x(t) = e^{-0.1t} \sin(t)$ .

`plot(t,x)`

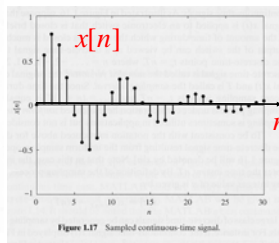


Figure 1.17 Sampled continuous-time signal.

`stem(n,x)`

## CT Signals: 1-D vs. N-D, Real vs. Complex

- ✓ • 1-D, real-valued, CT signal:  $x(t) \in \mathbb{R}, t \in \mathbb{R}$
- N-D, real-valued, CT signal:  $x(t) \in \mathbb{R}^N, t \in \mathbb{R}$
- ✓ • 1-D, complex-valued, CT signal:  $x(t) \in \mathbb{C}, t \in \mathbb{R}$
- N-D, complex-valued, CT signal:  $x(t) \in \mathbb{C}^N, t \in \mathbb{R}$

## DT Signals: 1-D vs. N-D, Real vs. Complex

- ✓ • 1-D, real-valued, DT signal:  $x[n] \in \mathbb{R}, n \in \mathbb{Z}$
- N-D, real-valued, DT signal:  $x[n] \in \mathbb{R}^N, n \in \mathbb{Z}$
- ✓ • 1-D, complex-valued, DT signal:  $x[n] \in \mathbb{C}, n \in \mathbb{Z}$
- N-D, complex-valued, DT signal:  $x[n] \in \mathbb{C}^N, n \in \mathbb{Z}$

## Digital Signals

- A DT signal whose values belong to a finite set or alphabet  $A = \{\alpha_1, \alpha_2, \dots, \alpha_N\}$  is called a **digital signal**
- Since computers work with **finite-precision arithmetic**, only digital signals can be numerically processed
- **Digital Signal Processing (DSP)**: ECE 464/564 (Liu) and ECE 567 (Lucchese)

### Digital Signals: 1-D vs. N-D

- 1-D, real-valued, digital signal:  $x[n] \in \mathbb{A}, n \in \mathbb{Z}$
- N-D, real-valued, digital signal:  $x[n] \in \mathbb{A}^N, n \in \mathbb{Z}$

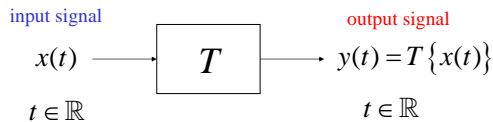
$$\mathbb{A} = \{\alpha_1, \alpha_2, \dots, \alpha_N\}$$

If  $\alpha_i \in \mathbb{R}$ , the digital signal is real, if instead at least one of the  $\alpha_i \in \mathbb{C}$ , the digital signal is complex

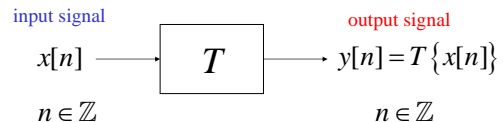
### Systems

- A **system** is any device that can **process signals** for analysis, synthesis, enhancement, format conversion, recording, transmission, etc.
- A system is usually mathematically defined by the equation(s) relating input to output signals (**I/O characterization**)
- A system may have single or multiple inputs and single or multiple outputs

### Block Diagram Representation of Single-Input Single-Output (SISO) CT Systems

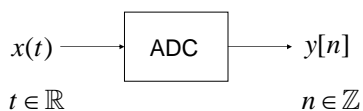


### Block Diagram Representation of Single-Input Single-Output (SISO) DT Systems



### A Hybrid SISO System: The Analog to Digital Converter (ADC)

Used to convert a CT (analog) signal into a digital signal



### Block Diagram Representation of Multiple-Input Multiple-Output (MIMO) CT Systems

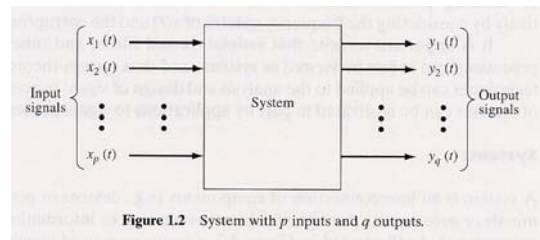
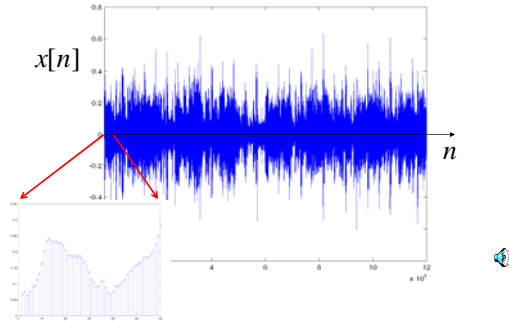
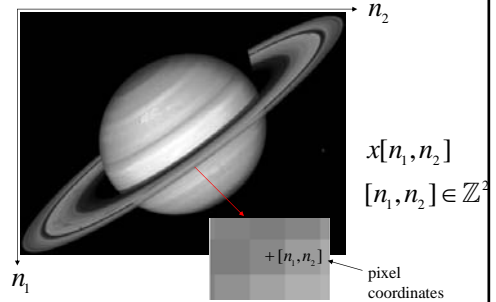


Figure 1.2 System with  $p$  inputs and  $q$  outputs.

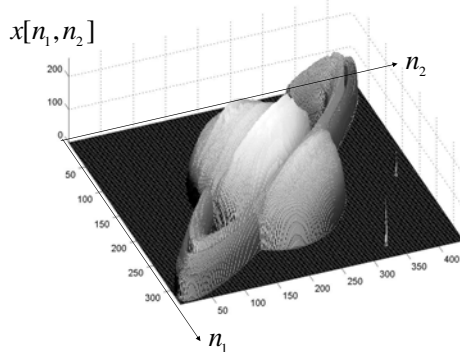
Example of 1-D, Real-Valued, Digital Signal: Digital Audio Signal



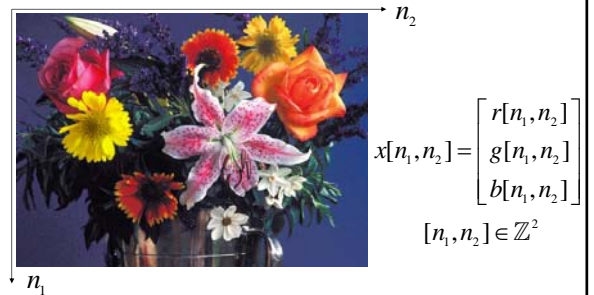
Example of 1-D, Real-Valued, Digital Signal with a 2-D Domain: A Digital Gray-Level Image



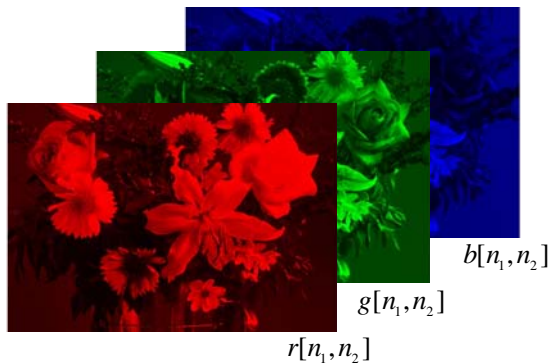
Digital Gray-Level Image: Cont'd



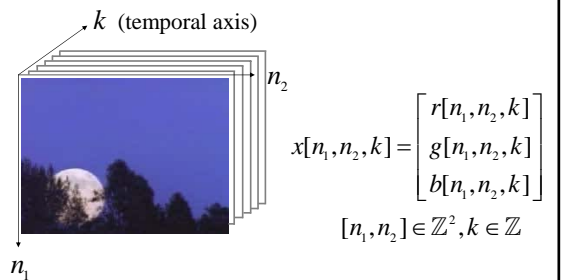
Example of 3-D, Real-Valued, Digital Signal with a 2-D Domain: A Digital Color Image



Digital Color Image: Cont'd



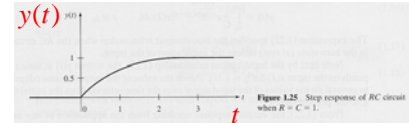
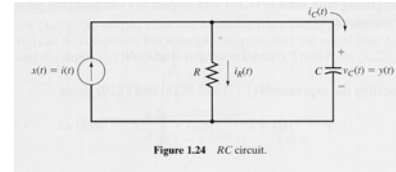
Example of 3-D, Real-Valued, Digital Signal with a 3-D Domain: A Digital Color Video Sequence



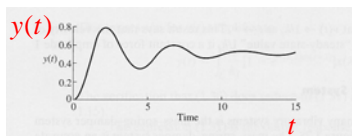
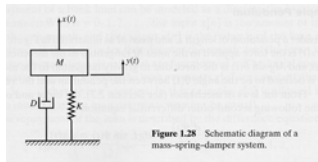
### Types of input/output representations considered

- Differential equation (or difference equation)
- The convolution model
- The transfer function representation (Fourier transform representation)

### Examples of 1-D, Real-Valued, CT Signals: Temporal Evolution of Currents and Voltages in Electrical Circuits

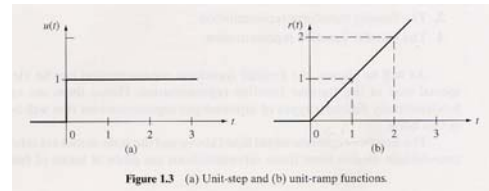


### Examples of 1-D, Real-Valued, CT Signals: Temporal Evolution of Some Physical Quantities in Mechanical Systems



### Continuous-Time (CT) Signals

- Unit-step function  $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$
- Unit-ramp function  $r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$



### Unit-Ramp and Unit-Step Functions: Some Properties

$$x(t)u(t) = \begin{cases} x(t), & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$r(t) = \int_{-\infty}^t u(\lambda) d\lambda$$

$$u(t) = \frac{dr(t)}{dt} \quad (\text{to the exception of } t = 0)$$

### The Unit Impulse

- A.k.a. the **delta function** or **Dirac distribution**
- It is defined by:

$$\delta(t) = 0, \quad t \neq 0$$

$$\int_{-\varepsilon}^{\varepsilon} \delta(\lambda) d\lambda = 1, \quad \forall \varepsilon > 0$$

- The value  $\delta(0)$  is not defined, in particular  $\delta(0) \neq \infty$

### The Unit Impulse: Graphical Interpretation

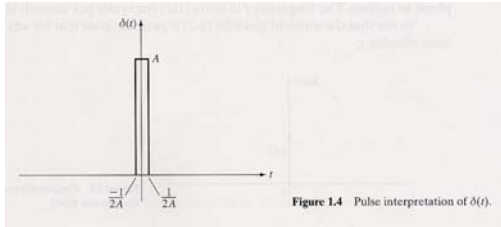


Figure 1.4 Pulse interpretation of  $\delta(t)$ .

A is a very large number

### The Scaled Impulse $K\delta(t)$

- If  $K \in \mathbb{R}$ ,  $K\delta(t)$  is the impulse with area  $K$ , i.e.,

$$K(t) = 0, \quad t \neq 0$$

$$\int_{-\varepsilon}^{\varepsilon} K\delta(\lambda) d\lambda = K, \quad \forall \varepsilon > 0$$

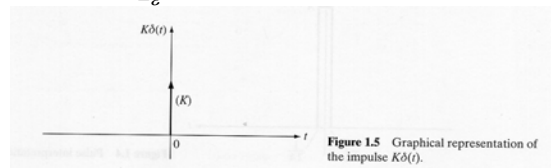


Figure 1.5 Graphical representation of the impulse  $K\delta(t)$ .

### Properties of the Delta Function

$$1) \quad u(t) = \int_{-\infty}^t \delta(\lambda) d\lambda$$

$$\forall t \text{ except } t = 0$$

$$2) \quad \int_{t_0-\varepsilon}^{t_0+\varepsilon} x(t)\delta(t-t_0)dt = x(t_0) \quad \forall \varepsilon > 0$$

(sifting property)

### Periodic Signals

- Definition: a signal  $x(t)$  is said to be periodic with period  $T$ , if

$$x(t+T) = x(t) \quad \forall t \in \mathbb{R}$$

- Notice that  $x(t)$  is also periodic with period  $qT$  where  $q$  is any positive integer
- $T$  is called the **fundamental period**

### Example: The Sinusoid

$$x(t) = A\cos(\omega t + \theta), \quad t \in \mathbb{R}$$

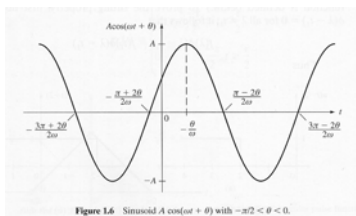


Figure 1.6 Sinusoid  $A\cos(\omega t + \theta)$  with  $-\pi/2 < \theta < 0$ .

$$\omega \text{ [rad/sec]}$$

$$\theta \text{ [rad]}$$

$$f = \frac{\omega}{2\pi} \text{ [1/sec] = [Hz]}$$

### Is the Sum of Periodic Signals Periodic?

- Let  $x_1(t)$  and  $x_2(t)$  be two periodic signals with periods  $T_1$  and  $T_2$ , respectively
- Then, the sum  $x_1(t) + x_2(t)$  is periodic only if the ratio  $T_1/T_2$  can be written as the ratio  $q/r$  of two integers  $q$  and  $r$
- In addition, if  $r$  and  $q$  are coprime, then  $T = rT_1$  is the fundamental period of  $x_1(t) + x_2(t)$

## Time-Shifted Signals

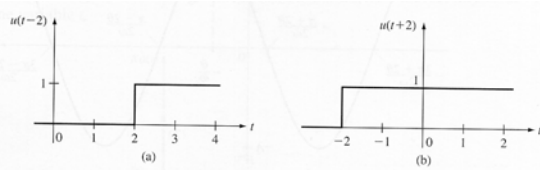
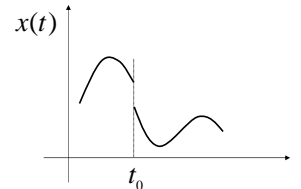


Figure 1.7 Two-second shifts of  $u(t)$ : (a) right shift; (b) left shift.

## Points of Discontinuity

- A continuous-time signal  $x(t)$  is said to be discontinuous at a point  $t_0$  if  $x(t_0^+) \neq x(t_0^-)$  where  $t_0^+ = t_0 + \varepsilon$  and  $t_0^- = t_0 - \varepsilon$ ,  $\varepsilon$  being a small positive number



## Continuous Signals

- A signal  $x(t)$  is continuous at the point  $t_0$  if  $x(t_0^+) = x(t_0^-)$
- If a signal  $x(t)$  is continuous at all points  $t$ ,  $x(t)$  is said to be a **continuous signal**

## Example of Continuous Signal: The Triangular Pulse Function

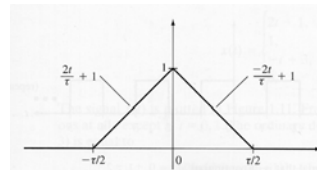


Figure 1.8 Triangular pulse function.

## Piecewise-Continuous Signals

- A signal  $x(t)$  is said to be piecewise continuous if it is continuous at all  $t$  except a finite or countably infinite collection of points  $t_i, i = 1, 2, 3, \dots$

## Example of Piecewise-Continuous Signal: The Rectangular Pulse Function

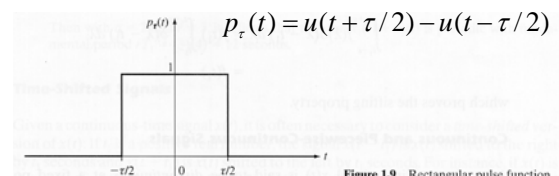


Figure 1.9 Rectangular pulse function.

### Another Example of Piecewise-Continuous Signal: The Pulse Train Function

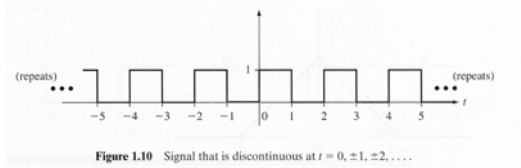


Figure 1.10 Signal that is discontinuous at  $t = 0, \pm 1, \pm 2, \dots$

### Derivative of a Continuous-Time Signal

- A signal  $x(t)$  is said to be **differentiable** at a point  $t_0$  if the quantity

$$\frac{x(t_0 + h) - x(t_0)}{h}$$

has limit as  $h \rightarrow 0$  independent of whether  $h$  approaches 0 from above ( $h > 0$ ) or from below ( $h < 0$ )

- If the limit exists,  $x(t)$  has a **derivative** at  $t_0$

$$\left. \frac{dx(t)}{dt} \right|_{t=t_0} = \lim_{h \rightarrow 0} \frac{x(t_0 + h) - x(t_0)}{h}$$

### Continuity and Differentiability

- In order for  $x(t)$  to be differentiable at a point  $t_0$ , it is necessary (but not sufficient) that  $x(t)$  be continuous at  $t_0$
- Continuous-time signals that are not continuous at all points (piecewise continuity) cannot be differentiable at all points

### Generalized Derivative

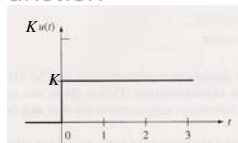
- However, piecewise-continuous signals may have a derivative in a generalized sense
- Suppose that  $x(t)$  is differentiable at all  $t$  except  $t = t_0$
- The **generalized derivative** of  $x(t)$  is defined to be

$$\frac{dx(t)}{dt} + [x(t_0^+) - x(t_0^-)]\delta(t - t_0)$$

ordinary derivative of  $x(t)$  at all  $t$  except  $t = t_0$

### Example: Generalized Derivative of the Step Function

- Define  $x(t) = Ku(t)$



- The ordinary derivative of  $x(t)$  is 0 at all points except  $t = 0$
- Therefore, the generalized derivative of  $x(t)$  is

$$K[u(0^+) - u(0^-)]\delta(t - 0) = K\delta(t)$$

### Another Example of Generalized Derivative

- Consider the function defined as

$$x(t) = \begin{cases} 2t + 1, & 0 \leq t < 1 \\ 1, & 1 \leq t < 2 \\ -t + 3, & 2 \leq t \leq 3 \\ 0, & \text{all other } t \end{cases}$$

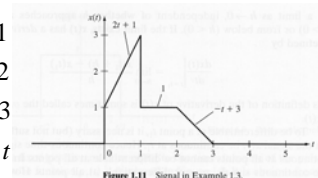


Figure 1.11 Signal in Example 1.3.



### Another Example of Generalized Derivative: Cont'd

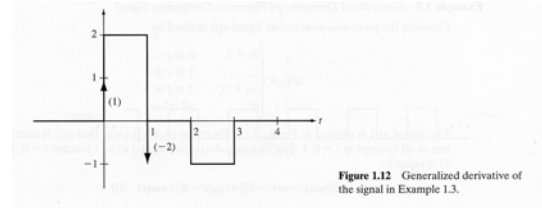
- The ordinary derivative of  $x(t)$ , at all  $t$  except  $t = 0, 1, 2, 3$  is

$$\frac{dx(t)}{dt} = 2[u(t) - u(t-1)] - [u(t-2) - u(t-3)]$$

- Its generalized derivative is

$$\frac{dx(t)}{dt} + \underbrace{[x(0^+) - x(0^-)]}_1 \delta(t) + \underbrace{[x(1^+) - x(1^-)]}_{-2} \delta(t-1)$$

### Another Example of Generalized Derivative: Cont'd



### Signals Defined Interval by Interval

- Consider the signal

$$x(t) = \begin{cases} x_1(t), & t_1 \leq t < t_2 \\ x_2(t), & t_2 \leq t < t_3 \\ x_3(t), & t \geq t_3 \end{cases}$$

- This signal can be expressed in terms of the unit-step function  $u(t)$  and its time-shifts as

$$x(t) = x_1(t)[u(t-t_1) - u(t-t_2)] + x_2(t)[u(t-t_2) - u(t-t_3)] + x_3(t)u(t-t_3), \quad t \geq t_1$$

### Signals Defined Interval by Interval: Cont'd

- By rearranging the terms, we can write

$$x(t) = f_1(t)u(t-t_1) + f_2(t)u(t-t_2) + f_3(t)u(t-t_3)$$

where

$$f_1(t) = x_1(t)$$

$$f_2(t) = x_2(t) - x_1(t)$$

$$f_3(t) = x_3(t) - x_2(t)$$

### Discrete-Time (DT) Signals

- A discrete-time signal is defined only over integer values
- We denote such a signal by

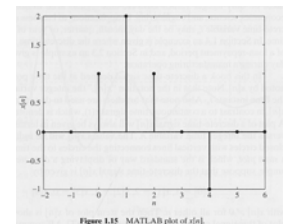
$$x[n], \quad n \in \mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$$

### Example: A Discrete-Time Signal Plotted with Matlab

- Suppose that

$$x[0]=1, \quad x[1]=2, \quad x[2]=1, \quad x[3]=0, \quad x[4]=-1$$

```
n=-2:6;
x=[0 0 1 2 1 0 -1 0 0];
stem(n,x)
xlabel('n')
ylabel('x[n]')
```



## Sampling

- Discrete-time signals are usually obtained by sampling continuous-time signals

$$x(t) \rightarrow \boxed{\text{Sampling}} \rightarrow x[n] = x(t)|_{t=nT} = x(nT)$$

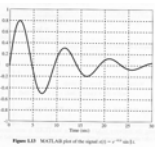


Figure 1.16: A plot of the signal  $x(t) = \cos(2\pi t/10)$  sampled at  $T = 10$  ms.

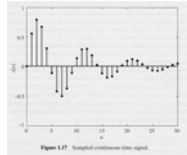


Figure 1.17: Sampled continuous-time signal.

## DT Step and Ramp Functions

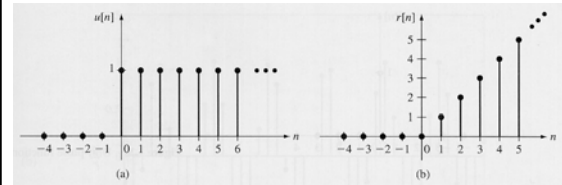


Figure 1.18: (a) Discrete-time unit step and (b) unit-ramp functions.

## DT Unit Pulse

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

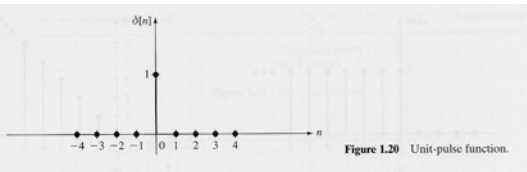


Figure 1.20: Unit-pulse function.

## Periodic DT Signals

- A DT signal  $x[n]$  is periodic if there exists a positive integer  $r$  such that

$$x[n+r] = x[n] \quad \forall n \in \mathbb{Z}$$

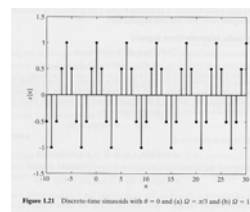
- $r$  is called the period of the signal
- The fundamental period is the smallest value of  $r$  for which the signal repeats

## Example: Periodic DT Signals

- Consider the signal  $x[n] = A \cos(\Omega n + \theta)$
- The signal is periodic if  $A \cos(\Omega(n+r) + \theta) = A \cos(\Omega n + \theta)$
- Recalling the periodicity of the cosine  $\cos(\alpha) = \cos(\alpha + 2k\pi)$   
 $x[n]$  is periodic if and only if there exists a positive integer  $r$  such that  $\Omega r = 2k\pi$  for some integer  $k$  or, equivalently, that the DT frequency  $\Omega$  is such that  $\Omega = 2k\pi/r$  for some positive integers  $k$  and  $r$

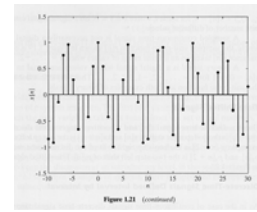
## Example: $x[n] = A \cos(\Omega n + \theta)$ for different values of $\Omega$

$$\Omega = \pi/3, \theta = 0$$



periodic signal with period  $r = 6$

$$\Omega = 1, \theta = 0$$



aperiodic signal (with periodic envelope)

### DT Rectangular Pulse

$$p_L[n] = \begin{cases} 1, & n = -(L-1)/2, \dots, -1, 0, 1, \dots, (L-1)/2 \\ 0, & \text{all other } n \end{cases}$$

( $L$  must be an odd integer)

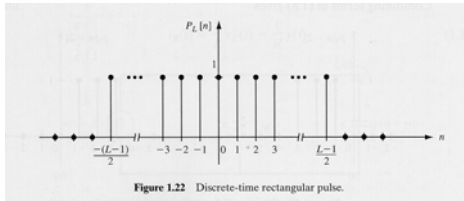


Figure 1.22 Discrete-time rectangular pulse.

### Digital Signals

- A **digital signal**  $x[n]$  is a DT signal whose values belong to a finite set or alphabet  $\{a_1, a_2, \dots, a_N\}$
- A CT signal can be converted into a digital signal by cascading the ideal sampler with a quantizer

### Time-Shifted Signals

- If  $x[n]$  is a DT signal and  $q$  is a positive integer  
 $x[n-q]$  is the  $q$ -step right shift of  $x[n]$   
 $x[n+q]$  is the  $q$ -step left shift of  $x[n]$

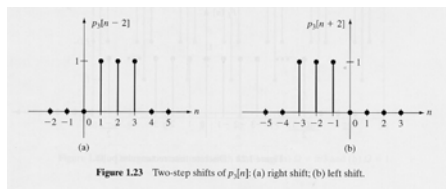


Figure 1.23 Two-step shifts of  $p_5[n]$ : (a) right shift; (b) left shift.

### Example of CT System: An RC Circuit

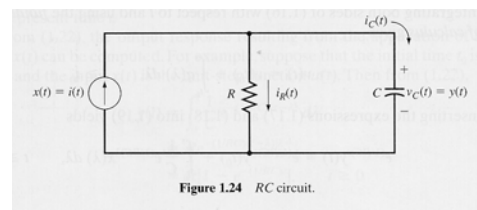


Figure 1.24 RC circuit.

Kirchhoff's current law:  $i_C(t) + i_R(t) = i(t)$

### RC Circuit: Cont'd

- The  $v$ - $i$  law for the capacitor is  

$$i_C(t) = C \frac{dv_C(t)}{dt} = C \frac{dy(t)}{dt}$$
- Whereas for the resistor it is  

$$i_R(t) = \frac{1}{R} v_C(t) = \frac{1}{R} y(t)$$

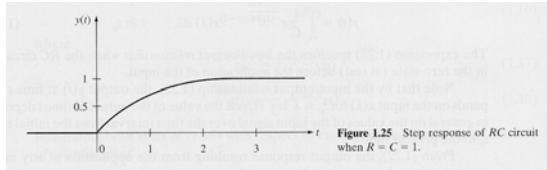
### RC Circuit: Cont'd

- **Constant-coefficient linear differential equation** describing the I/O relationship if the circuit

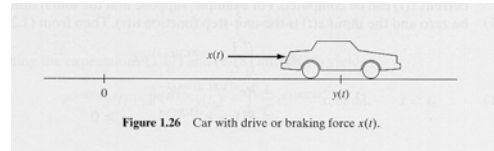
$$C \frac{dy(t)}{dt} + \frac{1}{R} y(t) = i(t) = x(t)$$

### RC Circuit: Cont'd

- Step response when  $R=C=1$



### Example of CT System: Car on a Level Surface



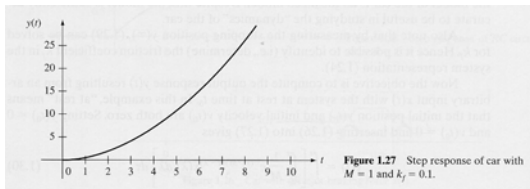
Newton's second law of motion:

$$M \frac{d^2 y(t)}{dt^2} + k_f \frac{dy(t)}{dt} = x(t)$$

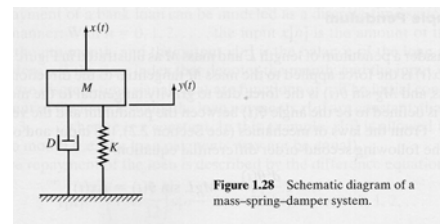
where  $x(t)$  is the drive or braking force applied to the car at time  $t$  and  $y(t)$  is the car's position at time  $t$

### Car on a Level Surface: Cont'd

- Step response when  $M=1$  and  $k_f = 0.1$



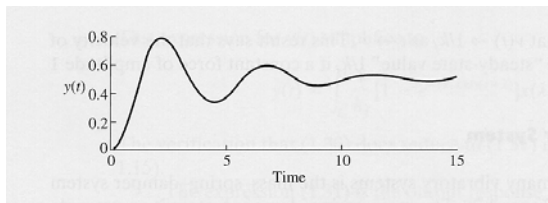
### Example of CT System: Mass-Spring-Damper System



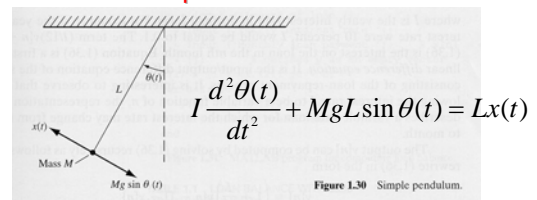
$$M \frac{d^2 y(t)}{dt^2} + D \frac{dy(t)}{dt} + Ky(t) = x(t)$$

### Mass-Spring-Damper System: Cont'd

- Step response when  $M=1$ ,  $K=2$ , and  $D=0.5$



### Example of CT System: Simple Pendulum



If  $\sin \theta(t) \approx \theta(t)$

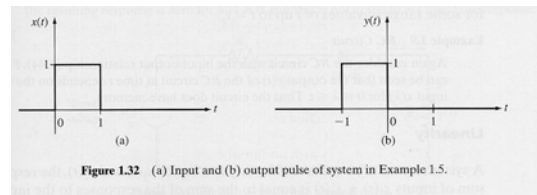
$$I \frac{d^2 \theta(t)}{dt^2} + MgL \theta(t) = Lx(t)$$

### Basic System Properties: Causality

- A system is said to be **causal** if, for any time  $t_1$ , the output response at time  $t_1$  resulting from input  $x(t)$  does not depend on values of the input for  $t > t_1$ .
- A system is said to be **noncausal** if it is not causal

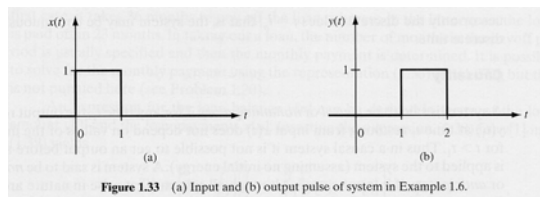
### Example: The Ideal Predictor

$$y(t) = x(t+1)$$



### Example: The Ideal Delay

$$y(t) = x(t-1)$$



### Memoryless Systems and Systems with Memory

- A causal system is **memoryless** or **static** if, for any time  $t_1$ , the value of the output at time  $t_1$  depends only on the value of the input at time  $t_1$ .
- A causal system that is not memoryless is said to have **memory**. A system has memory if the output at time  $t_1$  depends in general on the past values of the input  $x(t)$  for some range of values of  $t$  up to  $t = t_1$ .

### Examples

- Ideal Amplifier/Attenuator**

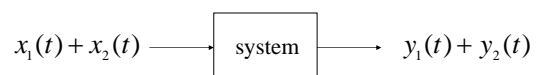
$$y(t) = Kx(t)$$

- RC Circuit**

$$y(t) = \frac{1}{C} \int_0^t e^{-(1/RC)(t-\tau)} x(\tau) d\tau, \quad t \geq 0$$

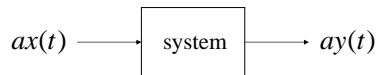
### Basic System Properties: Additive Systems

- A system is said to be **additive** if, for any two inputs  $x_1(t)$  and  $x_2(t)$ , the response to the sum of inputs  $x_1(t) + x_2(t)$  is equal to the sum of the responses to the inputs, assuming no initial energy before the application of the inputs



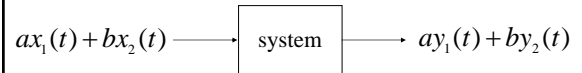
### Basic System Properties: Homogeneous Systems

- A system is said to be **homogeneous** if, for any input  $x(t)$  and any scalar  $a$ , the response to the input  $ax(t)$  is equal to  $a$  times the response to  $x(t)$ , assuming no energy before the application of the input



### Basic System Properties: Linearity

- A system is said to be **linear** if it is both additive and homogeneous



- A system that is not linear is said to be **nonlinear**

### Example of Nonlinear System: Circuit with a Diode

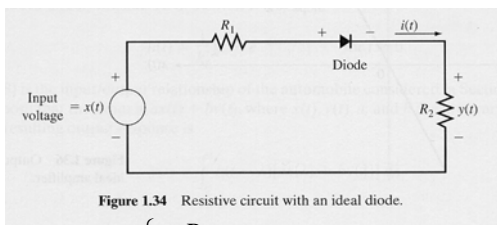


Figure 1.34 Resistive circuit with an ideal diode.

$$y(t) = \begin{cases} \frac{R_2}{R_1 + R_2} x(t), & \text{when } x(t) \geq 0 \\ 0, & \text{when } x(t) \leq 0 \end{cases}$$

### Example of Nonlinear System: Square-Law Device

$$y(t) = x^2(t)$$

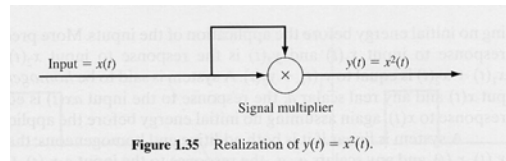


Figure 1.35 Realization of  $y(t) = x^2(t)$ .

### Example of Linear System: The Ideal Amplifier

$$y(t) = Kx(t)$$

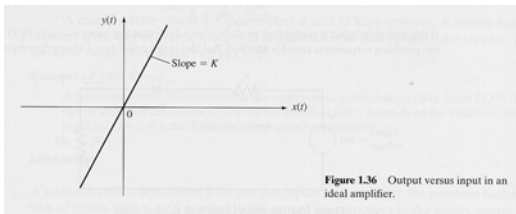


Figure 1.36 Output versus input in an ideal amplifier.

### Example of Linear System: A Real Amplifier

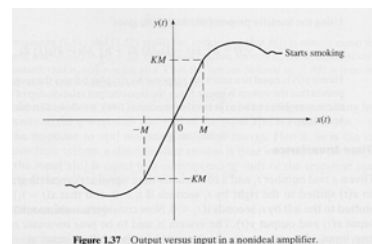
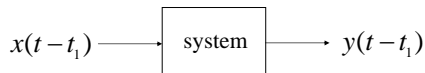


Figure 1.37 Output versus input in a nonideal amplifier.

### Basic System Properties: Time Invariance

- A system is said to be **time invariant** if, for any input  $x(t)$  and any time  $t_1$ , the response to the shifted input  $x(t - t_1)$  is equal to  $y(t - t_1)$  where  $y(t)$  is the response to  $x(t)$  with zero initial energy



- A system that is not time invariant is said to be **time varying** or **time variant**

### Examples of Time Varying Systems

- Amplifier with Time-Varying Gain**

$$y(t) = tx(t)$$

- First-Order System**

$$\dot{y}(t) + a(t)y(t) = bx(t)$$

### Basic System Properties: Finite Dimensionality

- Let  $x(t)$  and  $y(t)$  be the input and output of a CT system
- Let  $x^{(i)}(t)$  and  $y^{(i)}(t)$  denote their  $i$ -th derivatives
- The system is said to be **finite dimensional** or **lumped** if, for some positive integer  $N$  the  $N$ -th derivative of the output at time  $t$  is equal to a function of  $x^{(i)}(t)$  and  $y^{(i)}(t)$  at time  $t$  for  $0 \leq i \leq N - 1$

### Basic System Properties: Finite Dimensionality – Cont'd

- The  $N$ -th derivative of the output at time  $t$  may also depend on the  $i$ -th derivative of the input at time  $t$  for  $i \geq N$

$$y^{(N)}(t) = f(y(t), y^{(1)}(t), \dots, y^{(N-1)}(t), x(t), x^{(1)}(t), \dots, x^{(M)}(t), t)$$

- The integer  $N$  is called the **order** of the above **I/O differential equation** as well as the **order or dimension of the system** described by such equation

### Basic System Properties: Finite Dimensionality – Cont'd

- A CT system with memory is **infinite dimensional** if it is not finite dimensional, *i.e.*, if it is not possible to express the  $N$ -th derivative of the output in the form indicated above for some positive integer  $N$
- Example: System with Delay**

$$\frac{dy(t)}{dt} + ay(t-1) = x(t)$$

### DT Finite-Dimensional Systems

- Let  $x[n]$  and  $y[n]$  be the input and output of a DT system.
- The system is **finite dimensional** if, for some positive integer  $N$  and nonnegative integer  $M$ ,  $y[n]$  can be written in the form

$$y[n] = f(y[n-1], y[n-2], \dots, y[n-N], x[n], x[n-1], \dots, x[n-M], n)$$

- $N$  is called the **order** of the **I/O difference equation** as well as the **order or dimension of the system** described by such equation

### Basic System Properties: CT Linear Finite-Dimensional Systems

- If the N-th derivative of a CT system can be written in the form

$$y^{(N)}(t) = -\sum_{i=0}^{N-1} a_i(t) y^{(i)}(t) + \sum_{i=0}^M b_i(t) x^{(i)}(t)$$

then the system is both linear and finite dimensional

### Basic System Properties: DT Linear Finite-Dimensional Systems

- If the output of a DT system can be written in the form

$$y[n] = -\sum_{i=0}^{N-1} a_i(n) y[n-i] + \sum_{i=0}^M b_i(n) x[n-i]$$

then the system is both linear and finite dimensional

### Basic System Properties: Linear Time-Invariant Finite-Dimensional Systems

- For a CT system it must be

$$a_i(t) = a_i \quad \text{and} \quad b_i(t) = b_i \quad \forall i \text{ and } t \in \mathbb{R}$$

- And, similarly, for a DT system

$$a_i(n) = a_i \quad \text{and} \quad b_i(n) = b_i \quad \forall i \text{ and } n \in \mathbb{Z}$$

### About the Order of Differential and Difference Equations

- ✓ • Some authors define the order as  $N$
- Some as  $(M, N)$
- Some others as  $\max(M, N)$