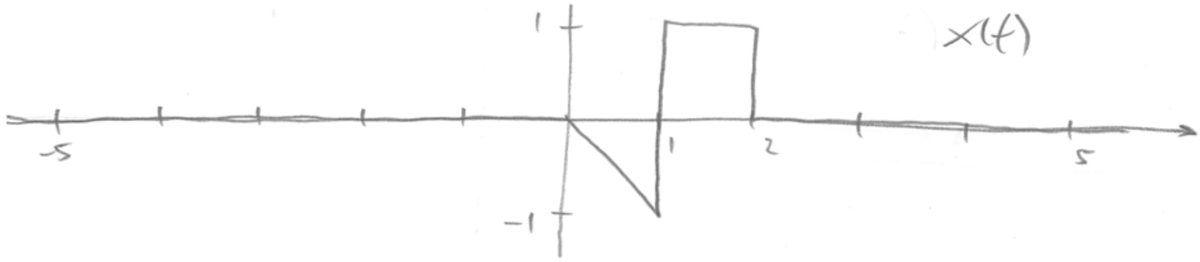
EEE2035F Homework assignment 1 solutions

① $x(t) = (t+1)u(t-1) - tu(t) - u(t-2)$

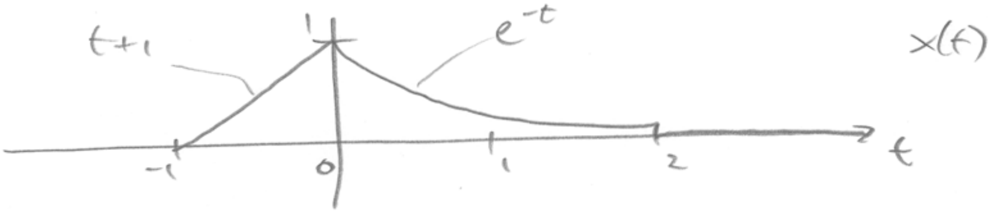
$$= tu(t-1) + u(t-1) - tu(t) - u(t-2)$$

$$= -t\underbrace{[u(t) - u(t-1)]} + \underbrace{[u(t-1) - u(t-2)]}$$

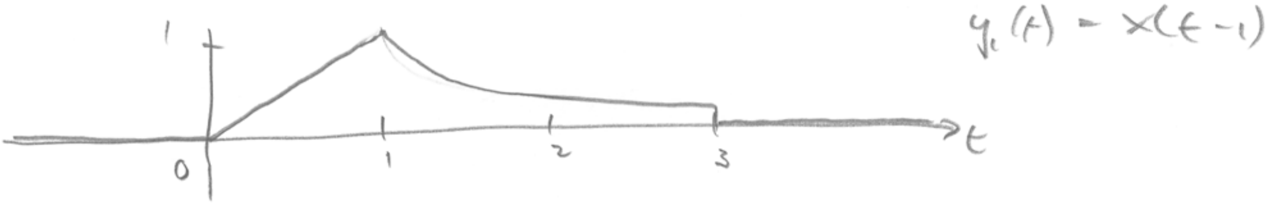
Therefore



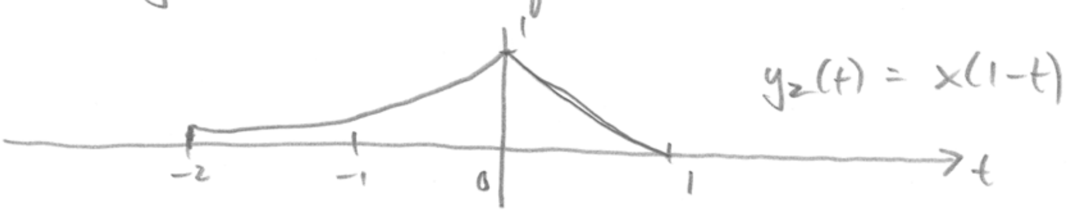
② a.



b. Origin moves to $t-1=0$ $(t=1)$ and no reflection:



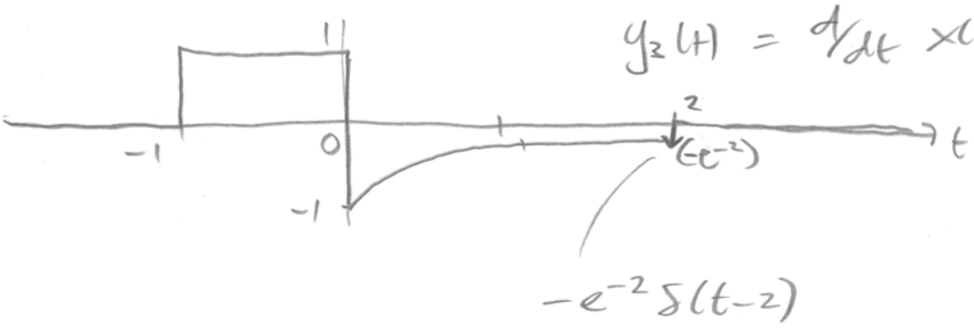
c. Minus sign in front of t, so reflection around origin. Origin moves to position where $1-t=0$ $(t=1)$



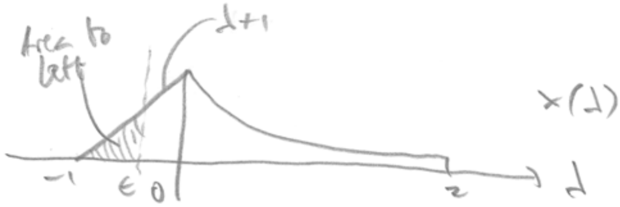
d. $\frac{d}{dt}(t+1) = 1$ $\qquad$ $\frac{d}{dt}e^{-t} = -e^{-t}$

Discontinuity of size $-e^{-2}$ at $t=2$.

$$y_3(t) = \frac{d}{dt}x(t)$$



f.

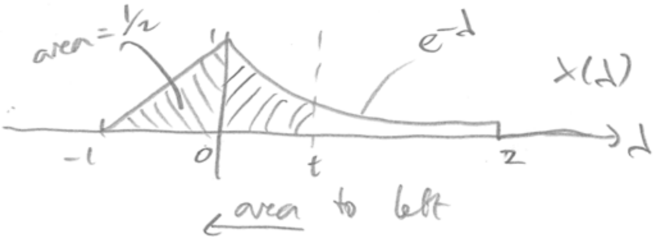


For $\boxed{t<-1}$, area to left is zero, so $\boxed{y_4(t) = 0}$

For $\boxed{-1<t<0}$, area to left is

$$\int_{-1}^{t}(\lambda+1)\,d\lambda = \left[\tfrac{1}{2}\lambda^2+\lambda\right]_{\lambda=-1}^{\lambda=t}$$

$$= \boxed{\tfrac{1}{2}t^2+t+\tfrac{1}{2} = y_4(t)}$$



For $\boxed{0<t<2}$, area to left is

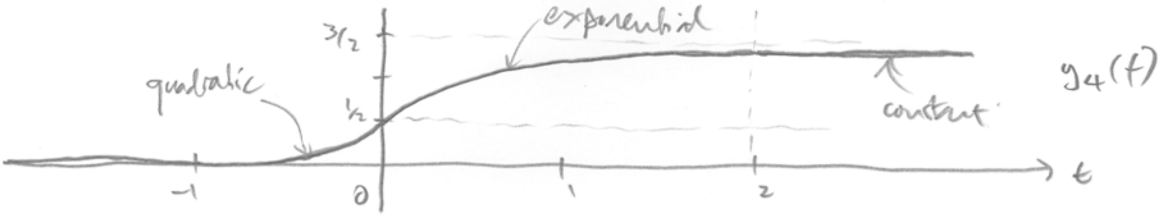
$$\tfrac{1}{2}+\int_{0}^{t}e^{-\lambda}\,d\lambda = \tfrac{1}{2}+\left[-e^{-\lambda}\right]_{\lambda=0}^{\lambda=t}$$

$$= \tfrac{1}{2}-e^{-t}+e^{0} = \boxed{\tfrac{1}{2}+(1-e^{-t}) = y_4(t)}$$

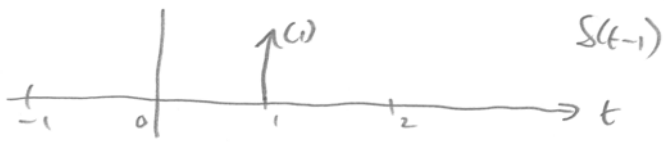
For $\boxed{t>2}$ area to left is

$$\boxed{\tfrac{1}{2}+(1-e^{-2}) = y_4(t)}$$

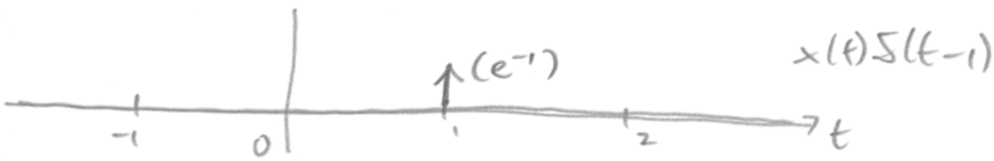
So



(3)



So



Thus $\displaystyle\int_{-\infty}^{\infty}x(t)\,\delta(t-1)\,dt = \int_{-\infty}^{\infty}e^{-1}\delta(t-1)\,dt = e^{-1}\underbrace{\int_{-\infty}^{\infty}\delta(t-1)\,dt}_{=1}$

$$= e^{-1} \quad (= x(1))$$