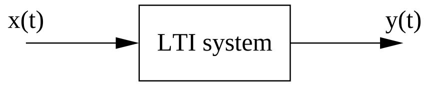
Linear time-invariant systems

Consider input/output relationships for a system:



Can characterise the system in three equivalent ways (in this course):

1. **LCCDEs**: $y^{(N)}(t) + \sum_{i=0}^{N-1} a_i y^{(i)}(t) = \sum_{i=0}^{M} b_i x^{(i)}(t)$
   System determined by coefficients $(a_0, a_1, \ldots, a_{N-1})$ and $(b_0, b_1, \ldots, b_M)$
   + Initial conditions (eg. $y(0), y^{(1)}, \ldots, y^{(N-1)}$)

2. **Convolution**: $y(t) = h(t) * x(t) \quad (= x(t) * h(t))$
   System determined by impulse response $h(t)$
   + Initial conditions (eg. initial rest)

3. **Frequency domain**: $Y(\omega) = H(\omega)X(\omega)$
   System determined by $H(\omega)$

Convolution

Definition:

$h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \quad (= \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau)$

Properties:

1. (*associative*): $[x(t) * v(t)] * w(t) = x(t) * [v(t) * w(t)]$
2. (*commutative*): $x(t) * v(t) = v(t) * x(t)$
3. (*distributive*): $x(t) * [v(t) + w(t)] = x(t) * v(t) + x(t) * w(t)$
4. (*identity*): $x(t) * \delta(t) = x(t)$

Also, if $x(t) * v(t) = w(t)$, then

5. (*shift*): $x(t-c) * v(t) = w(t-c)$
6. (*derivative*): $x^{(1)}(t) * v(t) = w^{(1)}(t)$
7. (*integral*): $x^{(-1)}(t) * v(t) = w^{(-1)}(t)$
8. (*shifted impulse*): $x(t) * \delta(t-c) = x(t-c)$

Fourier series

Any well-behaved periodic signal $x(t) = x(t+T)$ (for all t) has representation

$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\omega_0 kt} \qquad$ (with $\omega_0 = 2\pi/T$)

with coefficients (for any a)

$c_k = \frac{1}{T}\int_a^{a+T} x(t)e^{-j\omega_0 kt}dt$

Parsevals theorem: $\frac{1}{T}\int_a^{a+T} x^2(t)dt = \sum_{k=-\infty}^{\infty} |c_k|^2$

Fourier transform

For well-behaved $x(t)$ we have:

(*analysis*): $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

(*synthesis*): $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$

Denote this pair by: $x(t) \leftrightarrow X(\omega)$