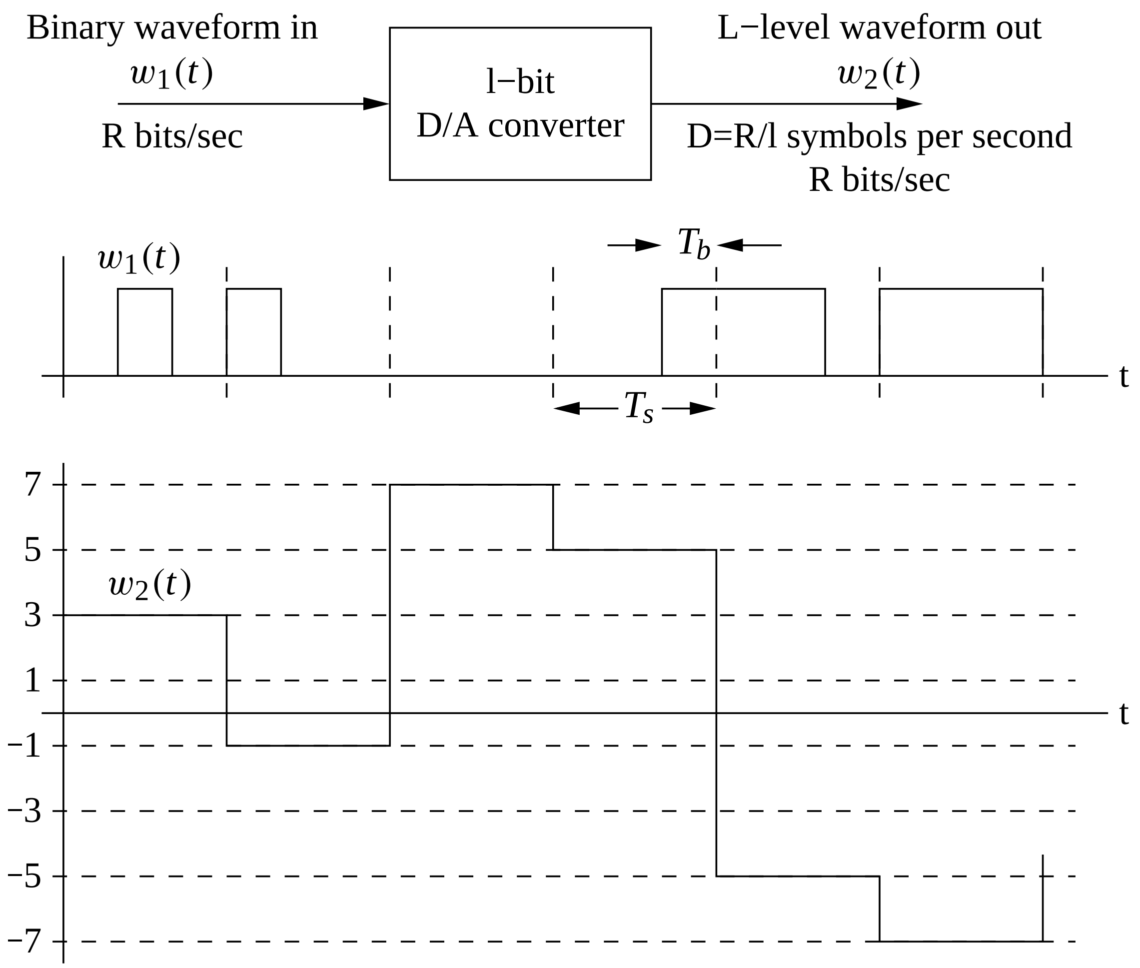
Multilevel signalling

The bandwidth required for transmission of *binary* digital waveforms may be very large. In particular, for a channel of bandwidth B Hz the Nyquist rate is $1/T = 2B$ symbols per second. In the case of binary signalling each symbol carries one bit of information, so the information rate is limited to $2B$ bits per second.

Clearly one can increase the information rate through a channel by increasing the bandwidth and the associated symbol rate. However, if the channel bandwidth is to remain fixed, then the only option is to increase the amount of information encoded in a symbol. M-ary signalling provides a means of achieving this:



If M is the number of distinct signal levels, then each symbol now carries $\log_2 M$ bits of information, and the overall information rate rises to $2B \log_2 M$. No additional bandwidth is required for this increase.

The increased information rate comes either at the expense of added transmitter power, or an increased error rate at the receiver. Consider M amplitude levels centred on zero, with $M = 2^l$. If A is the spacing between levels, then the levels are at

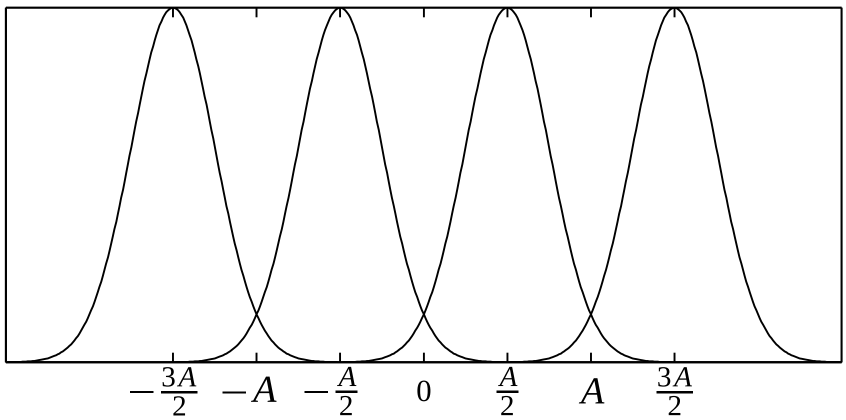
$$A_j = \pm\frac{A}{2}, \pm\frac{3A}{2}, \pm\frac{5A}{2}, \cdots, \frac{(M-1)A}{2}.$$

If all levels are equiprobable then the average signal power is

$$S = \frac{2}{M}\left\{\left(\frac{A}{2}\right)^2, \left(\frac{3A}{2}\right)^2 + \cdots \left(\frac{(M-1)A}{2}\right)^2\right\} = \frac{M^2-1}{3}\left(\frac{A}{2}\right)^2.$$

To maintain a constant spacing between levels, we therefore need to increase the transmitted power in proportion to the square of the number of levels.

At the same time, the spacing between levels directly determines the error rate at the receiver. To see this, consider the case of $M = 4$: in the event of additive Gaussian noise the PDFs at the receiver will be as follows:



As before, for equiprobable symbols the optimal thresholds lie midway between the levels.

Given that the actual level was at $-3A/2$, the probability of an error being made using the decision rule is

$$P_\epsilon|_{-3A/2} = \int_{A/2}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-y^2/(2\sigma^2)} = \mathrm{erfc}\left(\frac{A}{2\sigma}\right).$$

This is also the probability of an error being made given that the actual signal level is at $3A/2$. For signals at level $\pm A/2$, the probability of an incorrect decision is

$$P_\epsilon|_{\pm A/2} = 2\int_{A/2}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-y^2/(2\sigma^2)} = 2\ \mathrm{erfc}\left(\frac{A}{2\sigma}\right).$$

If all of these levels are equally likely, then the overall (average) probability of error is

$$\begin{aligned} P_w &= \frac{1}{4}\left(P_\epsilon|_{-3A/2} + P_\epsilon|_{-A/2} + P_\epsilon|_{A/2} + P_\epsilon|_{3A/2}\right) \\ &= 1/4(1+2+2+1)\ \mathrm{erfc}\left(\frac{A}{2\sigma}\right) = 3/2\ \mathrm{erfc}\left(\frac{A}{2\sigma}\right). \end{aligned}$$

It can easily be inferred that for M levels the overall probability of error is

$$P_w = \frac{2(M-1)}{M}\ \mathrm{erfc}\left(\frac{A}{2\sigma}\right).$$

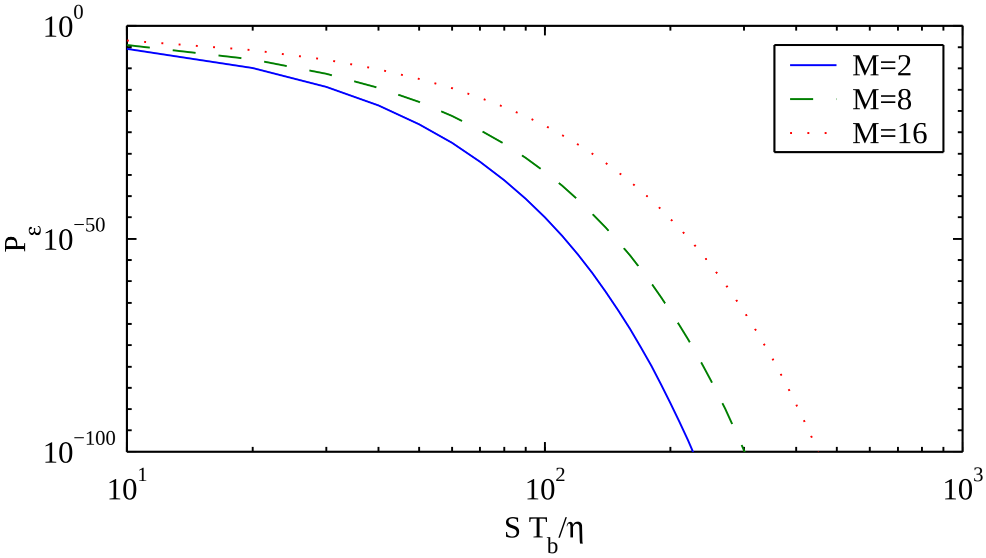
To obtain an expression for the error in terms of the power spectral density of the channel noise, we need to specify the channel and the receiver. Consider a matched filter designed for the signal transmitted at the $A/2$-level, and suppose that this signal has energy E. In the presence of the signal the matched filter output is then E, and the noise variance at the output is

$$\sigma^2 = E\eta/2.$$

When pulses with other amplitudes are received, the filter output value changes, but the noise variance remains the same. For input signals at each of the M levels, the matched filter has output values of

$$-(M-1)E, \ldots, -E, E, 3E, \ldots, (M-1)E.$$

It can be demonstrated that if there is no bandwidth constraint, then binary signalling has a lower error rate than M-ary signalling:



The main reason for using M-ary signalling is therefore when high information transfer rates are required over low bandwidth channels.

Note that *orthogonal* M-ary signalling can overcome this limitation. Here different signals are transmitted at different levels, with the difference being that the *shape* of the signals is also varied. This makes better use of available bandwidth, and error rates *decrease* as M increases. M-ary orthogonal signalling systems can approach the Shannon channel capacity as M tends to infinity.

Note that for M-ary signals it is meaningful to talk about the **baud rate**, which is the number of symbols transmitted per second.